

# The descriptive content of names as predicate modifiers

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**Abstract** In this paper I argue that descriptive content associated with a proper name can serve as a truth-conditionally relevant adjunct and be an additional contribution of the name to the truth-conditions. Definite descriptions *the so-and-so* associated by speakers with a proper name can be used as qualifying prepositional phrases *as so-and-so*, so sentences containing a proper name *NN is doing something* could be understood as *NN is doing something as NN* (which means *as so-and-so*). Used as an adjunct, the descriptive content of a proper name expresses the additional circumstances of an action (a manner, reason, goal, time or purpose) and constitute a part of a predicate. I argue that qualifying prepositional phrases should be analyzed as predicate modifiers and propose a formal representation of modified predicates. The additional truth-conditional relevance of the descriptive content of a proper name helps to explain the phenomenon of the substitution failure of coreferential names in simple sentences.

**Keywords** Names · Substitutivity · Modified predicates · Qualification · Adverbs · Adjuncts

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## 1 Introduction

In 2004 in Austria a scandal erupted around the holiday taken in the Tyrol region by the Belarusian president, Alexander Lukashenko. ‘Europe’s last dictator’ bypassed the EU sanctions which prohibited the entry of the Belarusian president to the European Union. But how was he able to do so despite being blacklisted? It transpired that Lukashenko also chaired the Belarusian Olympic Committee and some Austrian bankers managed to circumvent the ban on his entry to the EU by inviting him as the chairman of the Belarusian Olympic Committee. In the context of this unbelievable (but true) story, consider the two sentences below:

- (1) The President of Belarus is blacklisted
- (2) The chairman of the Belarusian Olympic Committee is not blacklisted

As we know, the President of Belarus and the chairman of the BOC is the same person. Intuitively both sentences could be true at the same time but, according to the standard definition of the satisfaction of formulas with a definite description in a modal logic given for example by Fitting and Mendelsohn (1998: 232, 254), these sentences are contradictory. Let ‘ $B$ ’ stand for ‘to be blacklisted’, ‘ $\iota x.P(x)$ ’ stand for ‘the present President of Belarus’ and ‘ $\iota x.C(x)$ ’ stand for ‘the present chairman of the Belarusian Olympic Committee’. According to standard definition,

- (1)  $\mathfrak{M}^{gwt} \models (\lambda y.B(y))(\iota x.P(x)) \text{ iff }$   
 $\mathfrak{M}^{g(y)wt} \models B(y)$ , where  $d = I_{(w,t)}^g(\iota x.P(x))$ .
- (2)  $\mathfrak{M}^{gwt} \models (\lambda y. \sim B(y))(\iota x.C(x)) \text{ iff }$   
 $\mathfrak{M}^{g(y)wt} \models \sim B(y)$ , where  $d = I_{(w,t)}^g(\iota x.C(x)) \text{ iff }$   
 $\mathfrak{M}^{g(y)wt} \not\models B(y)$ , where  $d = I_{(w,t)}^g(\iota x.C(x))$ .

As we know from the story above,  $I_{(w,t)}^g(\iota x.P(x)) = I_{(w,t)}^g(\iota x.C(x))$ , so sentences (1) and (2) are contradictory because one and the same person could not be blacklisted and not blacklisted at the same time.<sup>1</sup> Sentences such as (1) and (2) constitute an example of non-substitutivity between two coreferential expressions in simple sentences. Link (1983), Landman (1989) and Szabó (2003) drew attention to the fact that the phenomenon of lacking substitutivity is widespread and, apart from descriptions, concerns coreferential groups terms (*The Committee Puzzle*), plural terms (‘The judges/the hangmen are on strike’, Landman 1989: 724) and natural kinds terms (‘Water is often dirty, but  $H_2O$  is never dirty’, Szabó 2003: 387). Recently Saul (1997, 2007) noted that a substitution failure also occurs in simple sentences when a change from one coreferential proper name to another affects the truth-value of a sentence in an extensional context. Consider:

- (3) Cassius Clay was never beaten whereas Muhammad Ali lost five times.

<sup>1</sup> A scope of a negation is without importance here because it does not affect truth-conditions—assuming that the descriptions designate, formula (2) and its variant with a negation in wide scope have exactly the same truth-conditions.

Intuitively, (3) could be true: Muhammad Ali was never beaten when he fought as Cassius Clay and he lost five times when he fought as Muhammad Ali. The divergences between intuitions and formal truth conditions suggest that the role of descriptive content associated with proper names (and other terms) could not only be reference determining but this content could also be truth-conditionally relevant in some other way and be an additional contribution of a proper name into truth-conditions. This hypothesis explains why intuitive truth-conditions for sentences with proper names (and other terms) could differ from the truth-conditions of these sentences in a standard model. In this paper I will concern myself with the descriptive content of proper names and develop a hypothesis that it could behave as a truth-conditionally relevant adjunct and, as such, could modify a predicate (that is, could express truth-conditionally relevant circumstances of action named by the predicate). The main thesis of this paper is that identifying descriptions *the so-and-so* associated by speakers with a proper name could be used as qualifying prepositional phrases *as so-and-so*, so the sentences containing a proper name *NN is doing something* could be understood as *NN is doing something as NN* (which means *as so-and-so*).

The paper is structured in the following manner. In the next two sections I explain the notion of *as*-phrases modification and list its semantic properties. In Sect. 4 I propose a formal representation of *as*-phrases modification and, in Sect. 5, I briefly outline a way in which the puzzle of substitution failure of proper names in simple sentences could be solved with the help of this formalism. Section 6 contains concluding remarks and finally in Sect. 7 I present the formal machinery for predicate modifiers and prove some statements.

## 2 Qualifying prepositional *as*-phrases

Let us look again at (1), (2) and (3) sentences. It is very natural to paraphrase all of them using *as*-phrases: Lukashenko had a ban on visiting the EU *as* the President of Belarus but had no ban on visiting the EU *as* the chairman of BOC. Similarly the greatest boxer was never beaten when he fought *as* Cassius Clay and he lost five times when he fought *as* Muhammad Ali.<sup>2</sup> All the paraphrases contain the *as* preposition which, as with all prepositions, syntactically should be completed by a NP-phrase (Carnie 2006: 69). Such a NP-phrase could be very complex and contain explicitly expressed predicates or it could be represented by the anaphoric pronoun *such* (*as such*). The presence of the *such* pronoun in a paraphrase is evidence of

<sup>2</sup> Similarly all examples with terms other than proper names and descriptions could also be paraphrased with *as*-phrases: 'The judges are on strike as judges'/'The hangmen, as hangmen, are not on strike' (Landman 1989: 729–730); 'Water is often dirty but H<sub>2</sub>O *as such* is never dirty', 'The statue is made of copper but the copper *as such* isn't made of anything' (Szabó 2003: 388), 'Lex fears Superman as such', 'Lex fears Clark, not as such but as Superman' (Forbes 2006: 158, 159).

adjectival anaphora with a property-denoting expression taken as antecedent.<sup>3,4</sup> For example, if we paraphrase (1) as ‘The President of Belarus is blacklisted *as such*’ we naturally understand that the anaphoric pronoun *such* stands in this sentence for ‘the President of Belarus’. Similarly in cases of sentences containing an *as*-phrase and a proper name (as in Forbes’ example ‘Lex fears Clark, not *as such* but *as Superman*’) the proper name is understood as standing for a property, and that is why it can be replaced by the pronoun *such* which takes an adjective as an antecedent.

From the syntactical point of view *as*-prepositional phrases are adjuncts<sup>5</sup> and predicate modifiers (Carnie 2006: 164, 168).<sup>6</sup> In our Lukashenko story he was blacklisted *as* the President of Belarus but not *as* the chairman of BOC. Intuitively, the sentences (1) and (2) paraphrased as (1') and (2'),

(1') The President of Belarus is blacklisted *as the President of Belarus*

(2') The chairman of the BOC is not blacklisted *as the chairman of the BOC*

Could both be true. In such paraphrases a prepositional *as*-phrase modifies a predicate (Figs. 1, 2):

Sentences (1) and (2) are ambiguous and could be understood as stating that the referent of a description (Lukashenko) is (is not) blacklisted or that the referent of a description is (is not) blacklisted *as* the President of Belarus (*as* the BOC chairman). In the last case the sentences are understood in a way in which the descriptive content of a description modifies the predicate (Fig. 3).

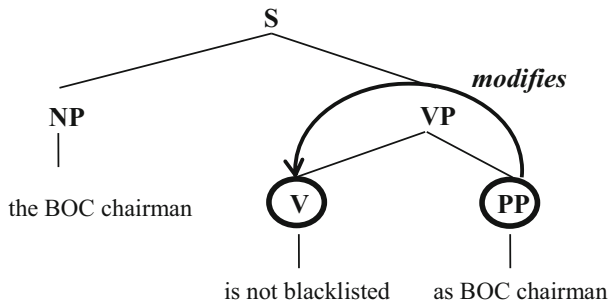
The codenoting descriptions ‘the BOC chairman’ and ‘the President of Belarus’ have different descriptive content which modifies the main predicate differently, and that is why the change of one description to another could affect the sentence truth-conditions. In effect, this possible change in truth-condition blocks the

<sup>3</sup> (Carlson 2003: 1231): ‘A wide variety of other anaphoric forms, beyond personal pronouns and temporal anaphora, make reference to an extensive array of other types of things. [...] Other forms take as antecedents phrases that are not NPs. [...] ‘such’ takes a modifier [...] If *intelligent* students attend college, *such* students usually do very well.’ (Landman and Morzycki 2003: 140–141): ‘*Such*, then, can be interpreted as a property of individuals that realize a contextually supplied kind.’ Landman (2006: 56): ‘As observed above, examples like the following [...] suggest an account of *such* as a property variable, as *such* appears to pick up the reference of a preceding adjective [...]’. The view that *such* is anaphoric to kinds is due to Carlson (1980: 230–236). Arguments supporting the claim that *such* behaves syntactically and semantically as an adjective and not as adjectival phrase could be found in Siegel (1994: 482) and in Wood (2002: 91).

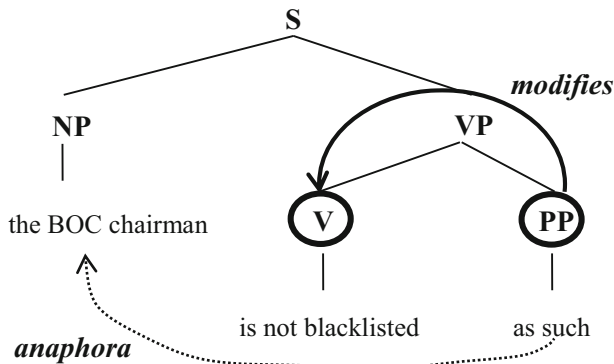
<sup>4</sup> Forbes (2006) proposed a solution of *The Superman Puzzle* based on a semantics of ‘as such’ phrases. According to him, in cases of intuitive substitution failure, simple sentences with proper names such as ‘Lex fears Superman’ should be understood as containing a covert prepositional phrase ‘Lex fears Superman as such’ (2006: 157–158). Forbes treats the *such* pronoun as a case of logophora (a special case of anaphora in which an expression serving as antecedent is taken itself as a referent of an anaphoric pronoun, 2006: 155, 158), but contrary to him I think that *such* is adjectivally anaphoric.

<sup>5</sup> Arguments in favor of this view are given by Szabó (2003: 395–397).

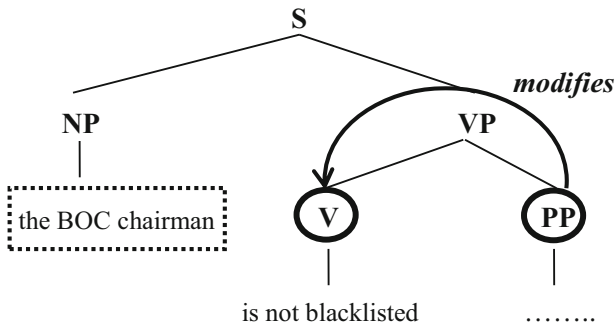
<sup>6</sup> By ‘modification’ I understand here a syntactical relation defined as follows (Carnie 2006: 85): ‘If an XP (that is, a phrase with some category X) modifies some head Y, then XP must be a sister to Y (i.e., a daughter to YP).’ Strictly speaking, modifying position for adjuncts on a tree is not to be a sister to N, V, A or P but to N', V', A' or P' (Carnie 2006: 162), which constitutes the main syntactical difference between adjuncts and complements. However I will leave aside this difference in tree position between complements and adjuncts (so I will present adjuncts in simplified manner as ‘sisters’ to ‘V’ on the trees below).



**Fig. 1** A prepositional *as*-phrase modification



**Fig. 2** An adjectival anaphora in a prepositional *as*-phrase



**Fig. 3** A modification by the descriptive content

substitution of descriptions. I think that we have the same phenomenon in the case of the substitution failure of coreferential proper names. But before I go further and present the semantics for modified predicates and outline how the descriptive content of proper names could be a predicate modifier, I need to mention the objections against predicate modification by *as*-phrases raised by Szabó (2003).

Szabó raised two objections against treating *as*-phrases as predicate modifiers (2003: 392). His *syntactic objection* has a general form and is raised against treating

*as*-phrases as modifiers of any sort. If *as*-phrases are modifiers it should be possible to iterate them but it is not the case (\*‘John earns \$50,000 as a judge as a janitor’). His *semantic objection* concerns an intuitive semantic connection between initial and modified predicates. Intuitively from ‘John was invited as a mathematician to the congress’ it is possible to conclude that ‘John was invited to the congress’ but for those who advocate predicate modification, an initial and modified predicates *are different*, so the connection between them is lost. Keeping in mind these objections, in the next two sections I will briefly present my proposal for treating *as*-phrases as predicate modifiers.

### 3 Semantic properties of prepositional *as*-phrases

Semantically, adverbs and adjuncts are used to express a manner, reason, goal, time, location, condition or purpose of action. Undoubtedly, prepositional phrases used as adjuncts are not adverbs but from the formal point of view both prepositional phrases and adverbs have been treated by philosophers in the same manner. In my proposal of semantics for *as*-phrases I will use some of the ideas of Romain Clark (1970) who proposed a logic for predicate modifiers.<sup>7</sup> The logic of predicate modifiers was intensively developed in the seventies<sup>8</sup> as an alternative to Davidsean (1967) semantics for action sentences. I will briefly present Clark’s idea in a nutshell. Consider Davidson’s well-known example:

(4) Sebastian strolled through the streets of Bologna at 2 a.m.

Davidson’s core idea was to add an additional argument place to action-predicates for an event-variable which could be bound by a quantifier. So, for example, sentences like (4) should have a logical form as in (4’) (1967: 167):

(4’)  $\exists_e(\text{Strolled}(S, e) \wedge \text{Through the streets of Bologna}(e) \wedge \text{At 2 a.m.}(e))$

As a result of such analysis it is easy to keep an intuitive entailment between (4’) and (4) which was one of the main advantages of the Davidsean account. Using standard predicate calculus it is impossible to conclude from ‘Sebastian strolled through the streets of Bologna at 2 a.m.’ that ‘Sebastian strolled’, because you need a 1-place predicate to represent ‘stroll’ and 3-place predicate to represent ‘stroll-through-at’, so in effect the semantic connection between the predicates is lost. Moreover, because adverbs and adjuncts are iterable, you can add any number of

<sup>7</sup> Besides the standard predicate modifiers Clark proposed the semantics for modifiers that he called ‘fictionalizers’ and ‘negators’ (1970: 329). The characteristic feature of such ‘falsifiers’ is that the intersection of two extensions—of initial predicate (‘Ming vase’) and of it being modified by a falsifier (‘a fake Ming vase’)—is an empty set. The proposition of analyzing expressions as ‘fake’, ‘mythical’, ‘simulated’ in a different way is due to Twardowski (1927). In this paper I will consider only standard modifiers and leave ‘falsifiers’ aside [more can be found in Poli (1991), Cocchiarella (2005), van der Schaar (2013)].

<sup>8</sup> See Lakoff (1970), Parsons (1970), Thomason (1971), Thomason and Stalnaker (1973), van Fraassen (1973), Richards (1976), Fulton (1979), Pörn (1982).

them to ‘stroll’ and in effect obtain plenty of semantically unconnected predicates, a somewhat undesirable consequence.

Clark (1970) proposed an alternative treatment of adverbs and prepositional phrases.<sup>9</sup> The core of his proposal is the idea that predicates could be built recursively out of  $n$ -place predicate constants by adding modifiers which have  $i$  places in total. So let us take ‘stroll’, for example. It is 1-place predicate. If you add the adverb ‘slowly’ to ‘stroll’ (getting ‘slowly stroll’) you would not increase the number of argument places. So ‘slowly’ is a 0-place modifier (as are many other adverbs). The extension of ‘slowly stroll’ is a subset of the extension of ‘stroll’ (Clark 1970: 325) and that is why you can infer from ‘Sebastian slowly strolled’ that ‘Sebastian strolled’ but not the other way around. This type of adverbial entailment failure is known as *Non-Entailment* (Davidson 1967; Katz 2008). Now take ‘at’ and ‘through’. Each of them are 1-place modifiers and if you add them to ‘stroll’ (getting ‘stroll-through-at’) you will increase the number of argument-places and will get a new 3-place predicate out of a 1-place initial one. You can infer from ‘Sebastian strolled through the streets of Bologna at 2 a.m.’ (Davidson 1967: 167) that ‘Sebastian strolled’ because the new 3-place predicate is connected with the initial 1-place predicate ‘stroll’ by a requirement that an object occupying the first place of the triple (Sebastian) should belong to the extension of ‘stroll’ (this type of entailment is called *Drop*). I will leave aside a syntactical definition of adjuncts [see (Carnie 2006: 162; 2008: 151)] together with all syntactic features specific for adjuncts such as iteration, reordering [called *Permutation*, see (Davidson 1967; Katz 2008)] and the ability of adjuncts to stand next to each other (Carnie 2006: 168). By ‘an adjunct predication’ I will understand predication fulfilling *Non-Entailment* and *Drop* semantic requirements [see (Davidson 1967; Katz 2008)].

Except for the mentioned adverbial entailment properties, sentences with *as*-phrases have one more type of entailment. I will use one of Szabó’s examples (2003: 406) to explain it. Consider: ‘John is rational as a chess-player’. Applying *Drop* we are able to infer from this that John is rational. But intuitively we can’t infer that John is rational *simpliciter*—he is rational in quite a specific way, that is, as a chess-player (compare a similar case with another prepositional phrase (Szabó 2003: 400): ‘I am happy about the news’. Intuitively, you can’t infer that I am happy *simpliciter*). From the conclusion you get after applying *Drop*, ‘X is  $\varphi$ ’, you cannot infer that ‘X is  $\varphi$  *simpliciter*’.<sup>10</sup> The *Drop* entailment seems to be unproblematic whereas conditions for the *simpliciter* entailment are not so easy to discern (cf. Szabó 2003: 403–404). It seems that it is possible to infer from *A as B is C* that *A is C simpliciter* when for any *D* (such that *A as D is C* is true) both *A as D is C* and *A as  $\sim D$  is C* are true (so there is no need to qualify, ‘She likes him as a philosopher and not as a philosopher—she simply likes him’) but this condition could be too strong, so I leave this entailment unsolved.

<sup>9</sup> A similar treatment can be found in (McConnell-Ginet 1982).

<sup>10</sup> Katz (2008: 229) takes cases like these as supplying the claim that state verbs (contrary to events verbs) could be restricted from *Drop*.

## 4 Modified predicates semantics

I propose to treat prepositional *as*-phrases (‘invited *as a mathematician*’) as 0-place predicate modifiers. Unlike other prepositional phrases, *as*-phrases do not increase the number of argument-places (‘invited’), and, unlike adverbs, they do not modify a predicate with all its argument places *as a whole*. Instead, they modify it on one argument-place only. Imagine a situation in which an object  $d$  is an agent of two simultaneous actions,  $A$  and  $B$ , but only one of these actions is such that  $d$  is doing it *as  $\varphi$* . Szabó attempted to give an appropriate truth-conditions for such a situation by means of a requirement that only one action from  $A$  and  $B$  was a part of  $d$ ’s state  $\varphi$ . I will preserve the spirit of such an intuition but instead of assuming a mereology of states and events I will use the inclusion relation between predicates’ extensions (we will see that these two ideas, whilst the same in spirit, will give different results, see footnote 14). Note that if you know that  $d$  is doing  $A$  and  $B$  and *is  $\varphi$* , you can’t infer that  $A$  or  $B$  is done by  $d$  *as  $\varphi$*  (by *Non-Entailment*). This entailment failure shows that the extension of a modified predicate *doing  $A$  as  $\varphi$*  although depending on the extensions of  $A$  and  $\varphi$  (by *Drop*), is not fully determined by them. As we see, the *Non-Entailment* property is the key property in solving the failure of the substitution puzzle. Now let me present the core of modified predicates analysis.

### 4.1 Syntax

The core idea is simple: intuitively, predicate modifiers make predicates from predicates (see Clark 1970: 320; Pörn 1982: 294; Thomason and Stalnaker 1973: 201; van Fraassen 1973: 104, 107). Formally predicates are built from predicate constants in a recursive way and, due to this, we will use the term ‘predicate’ to refer to all kinds of predicates—atomic predicates (predicate constants), predicate abstracts, modified predicates and modified predicate abstracts.

Let us start from modifiers. By modifier we will understand all predicates abstracted from an atomic formula or a conjunction of atomic formulas with one free variable, e.g.,  $\lambda x.Q(x)$ ,  $\lambda x.(P(x) \wedge Q(x))$ . I assume for simplicity that modifiers are subclass of predicates abstracts and have no free occurrence of variables. Now I will define how atomic predicates are modified:

**Definition 1** If  $Q$  is a  $n$ -place predicate constant and  $(\lambda x.\varphi)$  is a modifier then  $Q^i_{\lambda x.\varphi}$  is  $n$ -place predicate modified by  $(\lambda x.\varphi)$  on  $i$ th argument place of  $Q$  (where  $1 \leq i \leq n$ ).

Notation ‘ $\lambda x.\varphi$ ’ means that a predicate  $Q$  is modified by a predicate  $\lambda x.\varphi$  on  $i$ th

argument place:  $Q \left( y_1, \dots, \underbrace{y_i}_{\lambda x.\varphi}, \dots, y_n \right)$ . We can treat  $\varphi$  as complex adjective—it

can’t change the number of arguments of a predicate (as we remember, it is 0-place modifier). Let me give an example: *greet* is a two-place predicate,  $\varphi(x)$  is a formula with one free variable in which  $\varphi$  means ‘a host of a party’.  $greet^1_{\lambda x.\varphi}$ ,  $greet^2_{\lambda x.\varphi}$  are



predicates built via modification from the predicate constant *greet*; we read them ‘as a host of a party  $x$  greets  $y$ ’ (modification on the 1st argument place) and as ‘ $x$  greets  $y$  as a host of a party’ (modification on the 2nd argument place). We will use a simplifying convention and in case a modifier is a predicate abstracted from an atomic formula,  $P(x)$ , we will simply write ‘ $Q_P^i$ ’ instead of ‘ $Q_{\lambda x.P(x)}^i$ ’ and in case  $Q$  is 1-place predicate we will write ‘ $Q_P$ ’ instead of ‘ $Q_P^1$ ’.

It is a remarkable fact about sentences with prepositional phrases that they can often be structurally ambiguous when it is not clear what exactly a prepositional phrase modifies. Consider the example of such a sentence represented in Fig. 4 below<sup>11</sup>.

By modifying the same predicate *see* on different argument places we could avoid the structural ambiguity and emphasize the meaning which could not be emphasized by a conjunction of predicates. The sentence ‘Sherlock saw the man *and* was using binoculars’ is true in a situation in which Sherlock saw the man ‘with the unaided eye’ and was using binoculars (for example, scratching his knee with them). Intuitively the truth-conditions of the sentence describing the situation on the left picture differ from the truth-conditions of the sentence with a conjunction of two predicates.

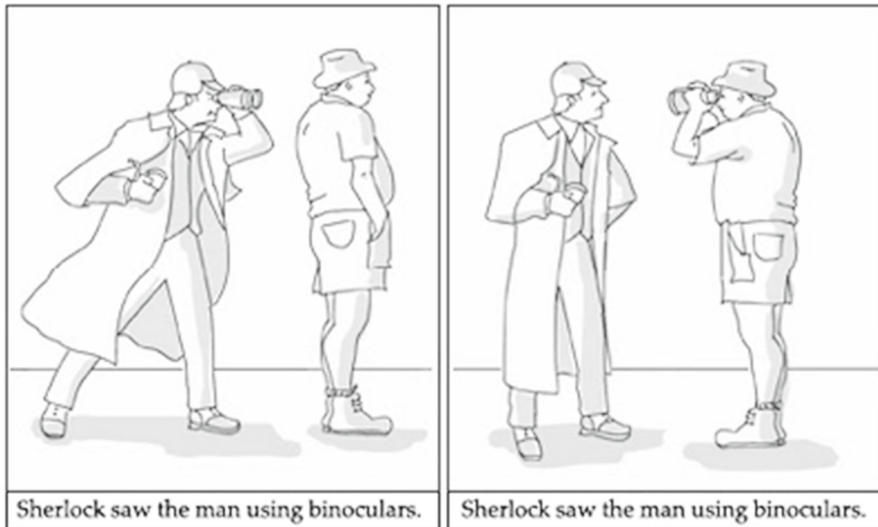
I allow predicate modification only on one ( $i$ th) argument place and do not say how to modify a predicate on other argument-places or on the same argument place again (I do not allow for iteration).<sup>12</sup> Nevertheless, the iteration of *as*-phrases is preserved in a limited form, because I allow that predicates abstracted from a conjunction of atomic formulas could be modifiers. Modifiers could be iterated by conjoining with *and* connective. In that way the Szabó example ‘\*John earns \$50,000 as a judge as a janitor’ is ungrammatical (in the same way as ‘\*John eats his steak with a fork with a knife’ because you cannot saturate the same argument position twice) but paraphrased with a conjunction (‘John earns \$50,000 as a judge and as a janitor’) becomes grammatical (this is my response to Szabó’s *syntactic objection*).

Now let me say a few words about the modification of predicate abstracts. Although a predicate could be abstracted from any formula, I will limit predicate abstracts which could be modified to predicates abstracted from atomic formulas and the negations of atomic formulas,  $\lambda x.Q(z_1, \dots, z_n)$  and  $\lambda x.\sim Q(z_1, \dots, z_n)$ . A modifier  $\lambda y.\psi$  modifies a predicate abstract on  $i$ th argument place of  $Q$  (written

<sup>11</sup> This picture is taken from von Fintel, Kai. 24.903 *Language and its Structure III: Semantics and Pragmatics*, Spring 2005. (MIT OpenCourseWare: Massachusetts Institute of Technology), <http://ocw.mit.edu/courses/linguistics-and-philosophy/24-903-language-and-its-structure-iii-semantics-and-pragmatics-spring-2005> (Accessed 26 May, 2014). License: Creative Commons BY-NC-SA.

<sup>12</sup> It seems that in natural language *as*-phrases could be iterated and could modify a modified predicate on the same argument place or on different argument place. Consider for example: ‘Teryl Austin has been confirmed as being hired as the Lions new defensive coordinator’ (example from ‘World News’, accessed 11 July, 2014; [http://article.wn.com/view/2014/01/18/Lions\\_hire\\_Teryl\\_Austin\\_as\\_defensive\\_coordinator\\_retain\\_8\\_fr/](http://article.wn.com/view/2014/01/18/Lions_hire_Teryl_Austin_as_defensive_coordinator_retain_8_fr/)). Intuitively, it is not only the case that Teryl Austin has been confirmed as being hired *and* as being the Lions new defensive coordinator (conjunction of modifiers), but he has been confirmed *as being hired as the Lions new defensive coordinator* (modification of a predicate *confirmed* by the already modified predicate *hired as a defensive coordinator*). Also a predicate could be modified on several argument-places, e.g., ‘As a cardiologist, I refuse to buy you a box of Havana cigars as a birthday gift’.

*Sherlock saw the man using binoculars [with binoculars]*



$see^1_{using\ binoculars}$

$see^2_{using\ binoculars}$

**Fig. 4** A modification of a predicate on different argument places

‘ $(\lambda x.\varphi)_{\lambda y.\psi}^i$ ’ in general notation).<sup>13</sup> Formulas with all kinds of predicates are built in a standard way.

## 4.2 Semantics

A few proposals concerning the semantics of modified predicates could be found in the literature and I will briefly mention one important moral which could be drawn from them before I explain my own proposal. As we said earlier, Clark (1970: 325) proposed treating a 0-place modifier such as ‘slowly’ in such a way that, when added to a predicate ‘slowly strolled’, it gives you a subset of ‘stroll’. But exactly which subset from a family of subsets of the predicate extension does it give? In order to answer this question it seems very natural to add a choice function to Clark’s account (e.g. Pörn 1982: 296). The role of a choice function would be to pick up one subset from the family of subsets of a predicate’s extension. Bas van Fraassen (1973) and James Fulton (1979) noted a serious problem with such an addition. I will use one of George Lakoff’s examples (1970: 5) to explain it. The problem appears when predicates have exactly the same extension. Consider a

<sup>13</sup> I will preserve the intuition that a modified predicate abstract  $(\lambda x.Q(z_1, \dots, z_n))_{\lambda y.\psi}^i$  and a predicate abstracted from a formula with a modified predicate  $(\lambda x.Q_{\lambda y.\psi}^i(z_1, \dots, z_n))$  are one and the same predicate (so you can take a modifier ‘in and out’ of a predicate abstract, see *Theorem I* in Sect. 8).

situation in which there are three guys, Albert ( $a$ ), Bernard ( $b$ ) and Clyde ( $c$ ), and each of them ate a hotdog and drank a beer. Let  $H$  stand for ‘to eat a hotdog’ and  $B$  stand for ‘to drink a beer’. These two predicates have the following interpretation:  $I(H) = \{a, b, c\}$ ,  $I(B) = \{a, b, c\}$ . ‘Slowly’ is a modifier and is supposed to be analyzed as a choice function on the family of subsets of extension of a predicate (minus the empty set). Because it is a function it would necessarily pick up exactly the same subsets in the case of coextensive predicates, so it would necessarily be so that exactly the same guys who slowly ate a hotdog also slowly drank a beer, which is unintuitive. That is why it seems reasonable to return to Clark’s idea (1970: 325) and let an interpretation  $I$  assign to a modified predicate  $MQ$  (formed from a predicate  $Q$  by prefixing a modifying operator  $M$  of degree zero) a subset of the extension assigned to  $Q$ :  $I(MQ) = (MQ)^{\text{mg}} \subseteq Q^{\text{mg}}$ . Subsets assigned to modified coextensive predicates could differ, for example:

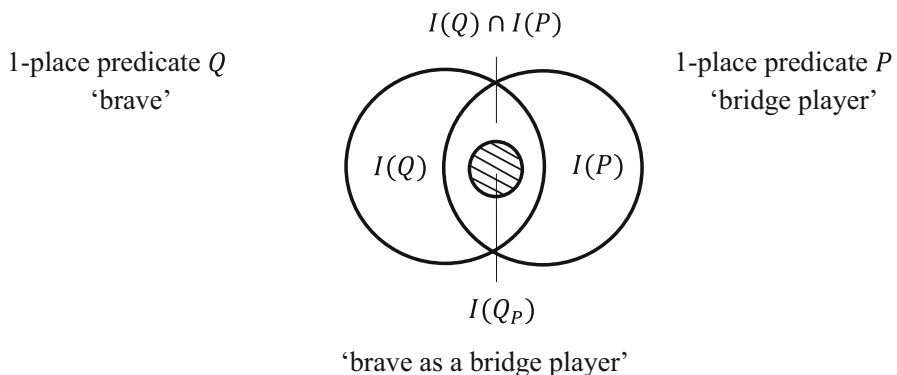
$$\begin{aligned} I(\text{Slowly } H) &= \{b, c\}; I(\text{Slowly } H) \subseteq I(H); \\ I(\text{Slowly } B) &= \{a\}; I(\text{Slowly } B) \subseteq I(B). \end{aligned}$$

Taking these arguments into account I defined an interpretation of modified predicates in the following way (for simplicity I will give first the definition of 1-place predicates  $Q$  and  $P$ ) (Fig. 5).

The definition of the interpretation of a modified predicate has the following general form:

**Definition 2** If  $Q$  is a  $n$ -place predicate constant,  $P$  is a 1-place predicate constant and  $x$  is a variable, then  $I_{\langle w, t \rangle}(Q_{\lambda x. P(x)}^i) \in \mathcal{P}(\{\langle d_1, \dots, d_i, \dots, d_n \rangle \in I_{\langle w, t \rangle}(Q) : d_i \in I_{\langle w, t \rangle}(P)\})$ .

A modified predicate is still a predicate—it is interpreted as a subset of the extension of the predicate being modified, a set of ordered  $n$ -tuples of objects of a domain. However, there is an additional condition: it should be such a set of  $n$ -tuples that every  $i$ th element in  $n$ -tuples fulfills the descriptive content  $\varphi$  with respect to the time and world of evaluation. This condition is needed to avoid the unintuitive



**Fig. 5** A modification of 1-place predicate

consequence that an  $n$ -tuple could belong to the extension of a modified predicate, despite the fact that the  $i$ th element in the  $n$ -tuple does not fulfill the descriptive content which modifies the predicate. In that way the interpretation of a modified predicate is related to the extension of the predicate being modified (we get *Drop* from *Definition 2* which is a response to Szabó's *semantic objection*).<sup>14</sup>

So let us return to the story with Lukashenko. It is possible to be the President of Belarus and to visit Austria without visiting this country as the President of Belarus because a person could belong to the extension of 'to visit Austria' ( $I(A)$ ) and to the extension of 'to be the President of Belarus' ( $I(P)$ ) but not to the extension of 'to visit Austria as the president of Belarus' ( $I(A_P)$ ). The extension of this last predicate is a *subset* of the extension of the set of people who are visiting Austria and are presidents of Belarus (singleton), and this subset could be empty (Fig. 6).

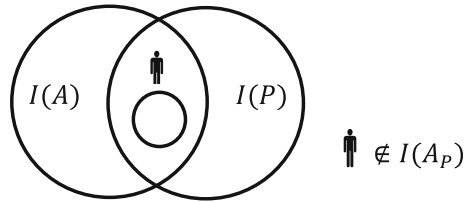
In such a way somebody could have a property  $\varphi$  but not a property ' $\varphi$  as  $\psi$ ' or could have a property ' $\varphi$  as  $\psi$ ' but not a property  $\varphi$  in any other way (to be  $\varphi$  only as  $\psi$ ). For example, the extension of the modified predicate 'to give an interview as a boxer' is a subset of people who gives an interview. Intuitively we could say about Madonna that she belongs to the set of people who gives an interview ( $GI$ ) but not to the subset of those who give interviews as a boxer ( $GI_{boxer}$ ),  $\P \in I(GI) \setminus I(GI_{boxer})$ . Intuitively, we could say about Michael Phelps that he took part in the Olympic Games ( $PO$ ) but only as a swimmer ( $PO_{swimmer}$ ), so we would not find him in any other subset of people taking part in the Olympic Games,  $\P \notin I(PO) \setminus I(PO_{swimmer})$ .<sup>15</sup> Also if somebody has a property  $\varphi$  and a property  $\psi$ , we could not conclude that that he has a property ' $\varphi$  as  $\psi$ ' (by *Non-Entailment*). The failure of making such a conclusion was noticed by Aristotle (*On Interpretation* XI 20b 35): 'Thus, again, whereas, if a man is both good and a shoemaker, we cannot combine the two propositions and say simply that he is a good shoemaker.'

Modifiers are closed under conjunction.

<sup>14</sup> Note that the interpretation of  $Q_P$  and  $P_Q$  predicates *could differ*. In case of  $I(Q_P)$  and  $I(P_Q)$  we have the same requirement that their extension should be a subset of  $I(Q) \cap I(P)$  set. Nothing restrains these sets from being different. But, assuming a mereology of states (as Szabó did), you would get quite an unintuitive result. Let me briefly explain. What does it mean on Szabó's (2003) account that somebody, say  $d$ , belongs to  $I(Q_P)$  and  $I(P_Q)$ ? This means [2003: 404 def. (51b)] that there are two states,  $s$  and  $s'$ , such that  $d$  is agent of both of them and  $P(s)$ ,  $Q(s')$  are true. Moreover, because  $d \in I(P_Q)$ , it should be so that  $s$  is a part of  $s'$ . In a similar way, because  $d \in I(Q_P)$ , it also should be so that  $s'$  is a part of  $s$ . This means that  $s$  and  $s'$  is one and the same state. I find this consequence quite unwelcome. Imagine that Jones is sentenced as a tax-dodger and as a prisoner he does not pay taxes. So his state  $s$  if being a prisoner is a part of his state  $s'$  of being a taxes nonpayer. In a similar way, his state  $s'$  is a part of state  $s$ . This means that his state  $s$  of being a prisoner and his state  $s'$  of being a nonpayer of taxes is one and the same state. This means in turn that every predicate true of  $s$  should also be true of  $s'$ . Imagine that Jones is a convinced anarchist and is unhappy to be imprisoned but is happy not to pay taxes. So he is both in a happy and an unhappy state. Assuming a mereology of states, Szabó was trying to avoid such a dilemma (2003: 400), but as we see, the dilemma still remains. Compare a similar example: 'As a suspect, Jones refuses to make a statement'/'As refusing to make a statement, Jones becomes a suspect'. On Szabó's account Jones's refusing-to-make-a-statement state and becoming-a-suspect state should be one and the same state but intuitively these states are two different states.

<sup>15</sup> I assume in this example that it is only possible to take part in the Olympic Games as an athlete of some kind.

**Fig. 6** ‘To visit Austria as the president of Belarus’



**Definition 3** If  $Q$  is a  $n$ -place predicate constant,  $x$  is a variable, and  $(\lambda x.\varphi)$ ,  $(\lambda x.\psi)$  are modifiers, then  $I_{\langle w,t \rangle}(Q_{\lambda x.(\varphi \wedge \psi)}^i) = I_{\langle w,t \rangle}(Q_{\lambda x.\varphi}^i) \cap I_{\langle w,t \rangle}(Q_{\lambda x.\psi}^i)$ .

In a similar way, modifiers could be closed under disjunction but I do not add such a definition. I need to say that the generalized logic of modified predicates seems to be a hard nut to crack. One of the puzzling things that comes to mind is a definition of a modifier's negation ('but now I am visiting your school *not as a police officer*'). You cannot define it simply as  $I_{\langle w,t \rangle}(Q_{\lambda x.\sim \varphi}^i) = I_{\langle w,t \rangle}(Q) \setminus I_{\langle w,t \rangle}(Q_{\lambda x.\varphi}^i)$ , because such a definition excludes the possibility for somebody who does  $Q$  as  $\varphi$  to do  $Q$  not as  $\varphi$  simultaneously ('I came to your school as a police officer but also I came to your school not as a police officer—as a father of one of the pupils'). You cannot define the modifier's negation also as  $I_{\langle w,t \rangle}(Q_{\lambda x.\sim \varphi}^i) = \mathcal{P}(I_{\langle w,t \rangle}(Q)) \setminus I_{\langle w,t \rangle}(Q_{\lambda x.\varphi}^i)$ , because you will have exactly the same consequence in case the extension of  $Q$  is a singleton (thanks to Leszek Wroński for discussion here). If you subtract any non-empty set from the family of subsets of  $Q$ , you will have the empty set, so it will be impossible for someone to do  $Q$  as  $\varphi$  and to do  $Q$  not as  $\varphi$  simultaneously. Maybe in order to define what does it mean that somebody does  $Q$  not as  $P$  it would be better to follow the intuition that it means that somebody does  $Q$  in **some other way**  $R$ , where  $R \neq P$ .

The other puzzling thing besides modifier negation is modification by a predicate abstracted from a formula with temporal operators ('She blessed him *as a future son-in law*' [as somebody who *would be* a son-in-law], 'As *a former police officer* [as somebody who *was* a police officer], he investigated quickly which kid scratched the car'). In a standard way a formula prefixed with a temporal operator is satisfied in a model iff the model satisfies the formula without the temporal operator with respect to a new time-parameter (shifted by the temporal operator). Such a definition is compositional—we drop the temporal operator and check if the formula without it is satisfied in a new time-parameter. It is clear that the compositionality is lost in the case of formulas with predicates modified by a predicate abstracted from a formula with a temporal operator. Intuitively, a guy who, *as a former police officer*, *now* investigates who scratched the car may have nothing in common with guys who *at some time in the past* investigated *as police officers* who scratched the car. You cannot simply, as before, shift a time parameter and check the truth-conditions of a formula without a temporal operator.

In my examples I have mentioned mainly *as*-phrases used as adjuncts of manner ('I will use the rest of the olive oil *as a base* for a salad dressing') but besides expressing a manner *as*-phrases could be used to express time ('Ann was fat *as a*

child'), reason ('As a firefighter, John was asked to help in the rescue action') and purpose ('They hired him as a launching engineer'). Maybe *as*-phrases could be used to express other characteristics of action or state named by a predicate and it is not clear if the analysis proposed here covers all types of use. For certain it doesn't cover *as*-phrases used as adjunct of comparison ('He is in his mid-forties but his mother still treats him *as a child*', 'He used a spoon *as a beer bottle opener*'). Understood in the most classical way, a comparison is an act of comparing thing A to thing B under respect C. Comparing A to B we are not saying that A *is* B (contrary to *Definition 2* requirements). Intuitively, when we say 'Your knife is blunt because you often use it as screwdriver and as a tent peg' we do not say that some particular knife *is* a screwdriver and *is* a tent peg (and, as a consequence, should belong to extensions of these predicates). All we are saying is that this particular knife is often used *in a similar way as* screwdrivers are used and *in a similar way as* tent pegs are used. So we can't use the semantics proposed here to analyze such examples because otherwise all objects such as a screwdriver, a tent peg, a knife, a sharpened ferule, etc. would belong to the extension of 'a screwdriver'/'a tent peg' predicates, which is unintuitive consequence. It seems to me that in case of *as*-phrases used as an adjunct of comparison we should give up the requirement that the object taking the *i*th argument-place should belong to the extension of a modifying predicate (in such a way *as*-phrases used as an adjunct of comparison may be analyzed in the same manner as adverbs in Clark's semantics).

## 5 Modification of a predicate by the descriptive content of a proper name

As I noted earlier, the phenomenon of substitution failure is quite widespread and seems to concern not only names but other corefering NPs, such as group terms, plural terms, natural kind terms and definite descriptions. All these expressions possess a descriptive content that determines an expression's reference.<sup>16</sup> As we have seen, sentences like (1) and (2), except of exemplifying the substitution failure, are ambiguous between modified and unmodified readings. This regular ambiguity

<sup>16</sup> An anonymous referee drew my attention to possible extension of the proposed account to indexicals. It seems, however, that there is no straightforward way to extend this account to sentences with indexicals, e.g. 'He (pointing at young Cassius Clay's photo in a newspaper) was never beaten but he (pointing at Muhammad Ali in another photo) was beaten five times'. Contrary to sentences with other coreferential NPs this is not a descriptive content of indexicals which is used as an adjunct (e.g. 'male individual' for 'he') because that is stable and thus could not modify the same predicate differently. The descriptive content which is used as an adjunct in this example is a contextually salient property (different in two acts of demonstration) possessed by a referent of an indexical, e.g. 'a boxer named [cassius clay]', 'a boxer named [muhammad ali]'. Firstly, it is unclear how one could extract such a property (should it be a description at all?), and secondly it is far from obvious how one should semantically connect such a property with an indexical. On the other hand, once the property is extracted and by 'He (pointing) was never beaten but he (pointing) was beaten five times' one understands something like 'He, as a boxer named [cassius clay], was never beaten but he, as a boxer named [muhammad ali] was beaten five times' or something like 'He was never beaten before 1964 but he was beaten five times after 1964', one can use the analysis of predicate modification presented here.

provides evidence that the descriptive content of a nominal phrase except for determining its extension functions as truth-conditionally relevant adjuncts and constitute an additional contribution to the truth-conditions (so I entirely agree with Szabó's proposal on this point). Regularity and truth-conditional relevance in its turn shows that the descriptive content is not suggested or implicated. I agree with Forbes (2006: 158) that the *as*-phrase invokes the mode of presentation connected with an expression and propose to treat this 'ways of giving' in a similar way as adverbs are treated—as predicates modifiers.

In this section I will explain how the descriptive content of proper names could modify a predicate and briefly sketch how the cases of substitution failure of coreferential proper names in simple sentences could be solved with the help of the formal semantics presented here (I will concern myself with proper names only and leave aside pseudonyms such as 'Superman' or 'Batman'. For a detailed solution of the puzzle see (Poller (under review a)). Consider our example (3) slightly modified as (3'):

(3') Cassius Clay was never beaten.

We have mixed intuitions about this sentence, because it seems true and untrue the same time. 'Cassius Clay' refers to Muhammad Ali and it was true about him that he never lost a fight when he fought as Cassius Clay but he lost five fights in his boxing career. But when we paraphrase (3') as (3'') or (3''') using *as*-phrase, both sentences seems true:

(3'') Cassius Clay was never beaten *as Cassius Clay*.

(3''') Cassius Clay was never beaten *as such*.

The reason why we have mixed intuitions about sentences from the puzzle such as (3') is because they are ambiguous between modified and unmodified readings. We could replace (3'') with (3'''), using the adjectivally anaphoric pronoun *such* which stands for a property. The possibility of such replacement supports the claim that proper name in the *as*-phrase in (3'') is understood as standing for a property, so the predicate in (3'') is modified not by a proper name but by a descriptive content of a proper name. So the idea at play behind the semantics of predicate modification by proper names is simple: the modifying content of a proper name  $n$  is a predicate  $\lambda x.\varphi$  abstracted from the formula  $\varphi$  of a definite description  $\iota y.\varphi$  connected with a proper name  $n$ . In order to write this idea as a definition I need to go through some syntactic definitions and to explain what it means for a description 'to be connected with a proper name  $n$ '.

Despite being a descriptivist (in my opinion speakers do associate definite descriptions with a proper name), I do not take the phenomenon of predicate modification by the descriptive content of proper names as an argument in favor of the descriptive theory of reference fixing. If you prefer another theory of names you could also accept the phenomenon of predicate modification by descriptive content of names—for example you can hold that descriptions are contained in mental files connected with names and used by speakers to modify predicates but nevertheless the descriptions do not semantically determine name's reference. Over the next two pages I will outline a way in which proper names could be formally represented. This formal representation is compatible with descriptive thesis about reference determination,



that is, with thesis that the reference of a name is semantically determined via satisfaction of descriptive properties. This means that the interpretation of a term which formally represents a proper name depends on the interpretation of a description which, in turn, depends on the interpretation of the predicates it contains. If you do not accept descriptivism you can find a way to connect sets of descriptions with names without an interpretation dependency. Note that the majority of model domains contain no speakers, and that is why I will leave aside all epistemic objections raised against descriptivism and concern on modal and circularity objections only. I am not defending descriptivism in this paper and epistemic objections stay unanswered here (I answered them in Poller (under review b)).

I represent proper names formally as special terms which I call ‘name-terms’. Such terms are rigid but semantically complex and receive their interpretation via a special sort of definite descriptions. I will briefly outline the idea behind such formal representation (a full version of the formal representation of proper names in accordance with the descriptive theory of reference can be found in (Poller 2014). Let me start from *iota*-terms. In a standard way they are built via applying a *i*-operator to a formula  $\varphi$  and designate with respect to a parameter of evaluation if there is only one object which fulfills  $\varphi$  in a set assigned to the evaluation parameter (otherwise *iota*-terms fail to designate, Fitting and Mendelsohn 1998: 254, 104). *Iota*-terms designate contingently with respect to possible worlds but if you add time as another point of evaluation, *iota*-terms would also designate contingently with respect to times. For example, take ‘the Pope’. It designates different people with respect to different times in our world (or fails to designate). This expression does not designate somebody in particular unless you add ‘present’ to it, getting ‘the present Pope’, or you express a time explicitly (e.g. ‘the Pope in 1967’). ‘The present Pope’, ‘the Pope in 1967’ expressions designate exactly one and *the same* person (if designates at all) with respect to a possible world and *any* time. So by ‘a definite description’ I understand *a special kind of iota-terms* of the form  $\iota x.[_i]\varphi$ , where ‘ $[_i]$ ’ is a notational variant of **then<sub>i</sub>** operator (‘true at  $t_i$ ’) taken after (Rini and Cresswell 2012). The time operator  $[_i]$  fixes a time of evaluation, so for any world  $w$ , time  $t$  and assignment  $g$   $I_{\langle w, t \rangle}^g(\iota x.[_i]\varphi) = I_{\langle w, t_i \rangle}^g(\iota x.\varphi)$ . In other words, a definite description  $\iota x.[_i]\varphi$  designates with respect to any time  $t$  the object designated by *iota*-term  $\iota x.\varphi$  with respect to time  $t_i$ . I will call definite descriptions  $\iota x.[_i]\varphi$  *actual with respect to  $t_i$* .

As we know from the previous section the account of modified predicates presented here is not general, so our most complicated modifier could be a predicate abstracted out of a conjunction of atomic formulas with one free variable. That is why I can use only *some* of definite descriptions  $\iota x.[_i]\varphi$ . To represent proper names formally we need a language  $\mathcal{L}$  with a set of distinguished predicates ( $N_1, N_2, N_3, \dots$ ) which we will read as ‘called  $\alpha$ ’, where ‘ $\alpha$ ’ is a string of sounds or an inscription [arguments supporting such view on verbs of naming could be found in (Geurts 1997: 326–328), see also (Matushansky 2008: 578, 580–581)]. I will use symbol ‘ $\iota x.\varphi$ ’ for *iota*-terms  $\iota x.\varphi$  with only one variable  $x$  which occurs free in  $\varphi$ . All descriptions  $\iota x.[_i]\varphi$  which we connect with a name-term have a form of  $\iota x.[_i](N_j(x) \wedge Q(x))$ , where ‘ $N_j$ ’ is a distinguished predicate and ‘ $Q$ ’ is a 1-place undistinguished, e.g. ‘the (present) president called [obama]’. A set of such descriptions will be called ‘ $\Gamma_{\mathcal{L}}$ ’ (see Sect. 7,

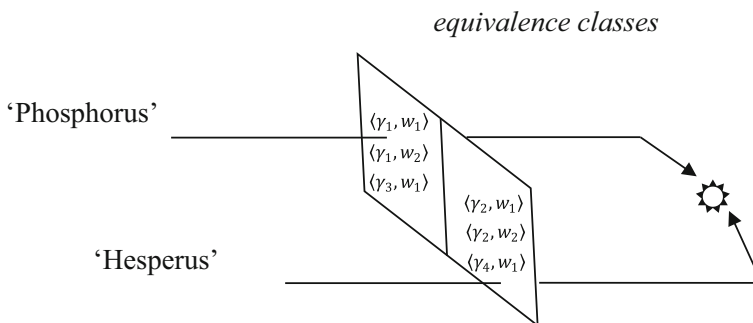


*Def.VI.S(a)*). We avoid Kripke's circularity argument by treating [obama] as a physical object (a sound or an inscription) which belongs to a model domain, not to a language. It is used as a mark to distinguish somebody (cf. Mill 1889/2011: 41), but the property of 'being called [obama]' is not sufficient to determine the reference because a lot of people are called so.

In order to avoid circularity we also need to be sure that the definite descriptions connected with a proper name contain no proper names. That is why we consider two languages,  $\mathcal{L}$  and  $\mathcal{L}^+$  ( $\mathcal{L} \subset \mathcal{L}^+$ ). Let me start from language  $\mathcal{L}$ . All terms it contains are variables and iota-terms (so there are no individual constants in  $\mathcal{L}$ ). We will use definite descriptions from  $\mathcal{L}$  to give interpretation of name-terms from  $\mathcal{L}^+$ . The idea behind connecting proper names with descriptions is simple: we let name-terms (formally representing names) designate through equivalence classes of descriptions which designate one and the same individual and contain one and the same predicate  $N_j$  (relation  $\mathbb{R}$ , see Sect. 7, *Def.VI.S(c)*). We need this last requirement in order to be able to formally distinguish formally two co-referring but distinct proper names (to represent them as two different name-terms). Because descriptions denote contingently we need to define an equivalence relation not on a set of descriptions  $\Gamma_{\mathcal{L}}$  but on a set of pairs containing a description and a world in which description designates (set  $\Delta$ , see Sect. 7, *Def.VI.S(b)*). So for example take two descriptions, 'the planet called [fosforus]', 'the planet called [hesperus]' (we name them  $\gamma_1, \gamma_2$  respectively). Both descriptions  $\gamma_1, \gamma_2$  designate in our world  $w$ , but pairs  $\langle \gamma_1, w \rangle, \langle \gamma_2, w \rangle$  will belong to different equivalence classes because  $\gamma_1$  contains predicate 'called [fosforus]' while  $\gamma_2$  contains different predicate 'called [hesperus]'. This idea is represented schematically in Fig. 7.

I won't go into formal details (full versions of definitions can be found in Sect. 7) and will instead just explain the key steps. In order to define an interpretation of a name-term  $n_i$  I need two functions—one which connects  $n_i$  with an equivalence class (function  $\mathbb{Q}^{\leq}$ , *Def.VI.S(e)*) and the other which takes an equivalence class and gives the object designated by every description in the class (function  $\mathbb{F}$ , *Def.VI.S(d)*). I presented this idea in Fig. 8.

The first function  $\mathbb{Q}^{\leq}$  for every name-term  $n_i$  gives an equivalence class,  $\mathbb{Q}^{\leq}(n_i) = \boxed{\text{equivalence class}}$  and the second function  $\mathbb{F}$  for every equivalence class



**Fig. 7** Different equivalence classes connected with co-referring names



**Fig. 8** An interpretation of a name-term  $ni$

gives an object designated by every description in that class,  $\mathbb{F}(\boxed{\text{equivalence class}}) = \odot$ . Now, letting a model  $\mathfrak{M}^{\leq}$  be a model for  $\mathcal{L}^+$  we can define an interpretation of a name-term in it as follows:

$$I_{\langle w, t \rangle}^{\leq}(n_i) = \mathbb{F}(\mathbb{Q}^{\leq}(n_i)).$$

In effect I have exactly what the description theory of reference postulates: new name-terms refer to objects via definite descriptions and that definite descriptions are used only to fix a reference and are not synonymous with name-terms. Such terms are obstinately rigid and not sensitive to the scope differences of temporal and modal operators in formulas without predicate modification by the descriptive content of a term (cf. Poller 2014, for proofs see also Poller 2014).

Now I will return to the question of modification. I let name-terms occupy an argument position of predicate abstracts only,  $(\lambda x. \varphi)(n)$ . Let me add some syntactic definitions (I will present them here in a simplified manner, see *Def. V.R13, R17, R18, R22* in Sect. 7):

## 5.1 Syntax

**Definition 4** If  $n$  is a name-term, then  $n$  is a modifier;

**Definition 5** If  $(\lambda x. Q(y_1, \dots, y_m))$ ,  $(\lambda x. \sim Q(y_1, \dots, y_m))$  are predicate abstracts and  $n$  is a naming term, then  $(\lambda x. Q(y_1, \dots, y_m))_n^i$ ,  $(\lambda x. \sim Q(y_1, \dots, y_m))_n^i$  are predicates abstracts modified by  $n$  on  $i$ th argument-place of  $Q$  (where  $1 \leq i \leq m$ );

**Definition 6** If  $(\lambda x. \varphi)_n^i$  is a modified predicate abstract and  $n$  is a name-term, then  $(\lambda x. \varphi)_n^i(n)$  is a formula.

Note that in case a predicate abstract is modified by a name-term, such a predicate and a name-term could form a formula iff the name-term occupying an argument place is the same as modifying name-term. As I noticed earlier, sentences as (3') could be paraphrased as (3'') with the anaphoric pronoun *such* (cf. 'Lex fears Superman *as such*', Forbes 2006: 158). Treating this case of anaphora in the most classical way, namely as a phenomenon of the interpretation dependence of occurrence of one expression on the interpretation of an occurrence of another expression, it is natural to suppose that the anaphoric pronoun takes as an antecedent an occurrence of the expression explicitly expressed in the same sentence (a proper name) which is the most salient occurrence of an expression.

Now I will return to the semantics of predicate modification by the descriptive content of a proper name. As I said earlier, the idea is quite simple: a formula

$(\lambda x.\varphi)_n^i(n)$  is satisfied in a model with respect to a world  $w$  and a time  $t$  iff there is a description  $!y.[_j]\psi$  in the set of descriptions for the term  $n$  and the world  $w$  such that the model satisfies  $(\lambda x.\varphi)_{\lambda y.\psi}^i(n)$  with respect to  $\langle w, t \rangle$ . In this definition I am trying to encapsulate the following idea: if we say that NN is doing something as NN, we mean by this that there is a (unspecified) way of describing NN such that NN is doing something in that way. All I have to do now is to explain what is the set of descriptions for the term  $n$  and the world  $w$  (I drop time-parameter  $t$  because, as I have said earlier, by a definite description I understand a *iota-term*  $\iota x.[_i]\varphi$  with a fixed time-parameter). What we have now is a connection between name-terms and equivalence classes of description-world pairs provided by  $\mathbb{Q}^{\leq}$  function, for example:

$$\mathbb{Q}^{\leq}(n) = \boxed{\begin{array}{c} \text{equivalence class} \\ \langle \gamma_i, w_1 \rangle \\ \langle \gamma_j, w_1 \rangle \\ \langle \gamma_k, w_2 \rangle \\ \langle \gamma_l, w_2 \rangle \end{array}}.$$

We will evaluate formulas with predicates modified by a descriptive content of proper names with respect to a possible world  $w$  and a time  $t$ . With respect to  $w$  we want to take into account only those descriptions which designate in  $w$ . That is why we need a function (let us call it  $h^{\leq}$ ) which for a term  $n$  and a world  $w$  returns a ‘smaller’ equivalence class of those descriptions which denote in  $w$  and is undefined in case there is no such class (see Sect. 7, *Def.VI.S(f)*, *Def.VI.S(g)*). For example, for a term  $n$  and world  $w_1$   $h^{\leq}$  gives you the following set:

$$h^{\leq}(n, w_1) = \boxed{\begin{array}{c} \text{‘smaller’} \\ \text{equivalence class} \\ \langle \gamma_i, w_1 \rangle \\ \langle \gamma_j, w_1 \rangle \end{array}}.$$

Now to have a set of descriptions (not pairs of descriptions and world) we take the first projection  $\pi_1$  of  $h^{\leq}(n, w_1)$  which gives a set of the first elements from every pair in a class:

$$\pi_1(h^{\leq}(n, w_1)) = \{\gamma_i, \gamma_j\}.$$

Now we can define a modification of a predicate by a descriptive content of a proper name as follows:

**Definition 7**  $\mathfrak{M}^{\leq gwt_j} \models (\lambda x.\varphi)_n^i(n)$  iff there is a description  $!y.[_j]\psi \in \pi_1(h^{\leq}(n, w))$ , such that  $\mathfrak{M}^{\leq gwt_j} \models (\lambda x.\varphi)_{\lambda y.\psi}^i(n)$ .

There are two interesting consequences for this definition. Note that although a name-term  $n$  is obstinately rigid (it refers to the same thing regardless of changes of time and world-parameters), its modifying descriptive content  $(\lambda y.\psi)$  obtained from a definite description  $!y.[_j]\psi$  by dropping fixing-time operator  $[_j]$  changes with respect to time and world of evaluation. This means that—while the name-term is not sensitive to the

scope differences of modal and temporal operators—its descriptive content *is sensitive*. In *Definition 7* we step from a formula  $(\lambda x. \varphi)_n^i(n)$  to a formula  $(\lambda x. \varphi)_{\lambda y. \psi}^i(n)$  and this last formula is satisfied in a standard way, when the referent of  $n$  belongs to the extension of the predicate  $(\lambda x. \varphi)_{\lambda y. \psi}^i$  regardless of satisfying any descriptive content. All the term's descriptive content does here is pick up the reference, and it is inessential in what way it does so because all you need for truth-conditions is just the referent itself and a property named by a predicate. Therefore, it is without any significance if the descriptive content of the name-term lies within or beyond the scope of a modal or temporal operator—nothing depends on a change in content. But when you want the descriptive content to express the additional circumstances of an action that should be taken into account you make the descriptive content a part of the predicate. The name-term is still not sensitive to points of evaluation but the predicate modified by the descriptive content of a term *is sensitive*. When we say that NN is doing something *as NN* we understand by it that NN is doing something in a descriptive way  $\psi$  *actual with respect to a time (and a world) of evaluation*. So by saying (3) ('Cassius Clay was never beaten whereas Muhammad Ali lost five times') we convey the idea that the greatest boxer never lost a fight during the period of time when he was a boxer actually called 'Cassius Clay' (and he lost five times after changing his name to 'Muhammad Ali'). What important conclusion could possibly be drawn from name-term's rigidity and the sensitivity of predicates modified by the name's descriptive content to points of evaluation? Perhaps in the case of proper names it is wrong to equate their rigidity with regards to the sameness of the truth-conditions of the two readings with wide and narrow scopes of sentences containing the term and a temporal and a modal operator.<sup>17</sup> For example, take a sentence  $(\lambda x. Q(x))_n^i(n)$  and the possibility operator  $\diamond$ . Applied in a wide-scope reading,  $\diamond(\lambda x. Q(x))_n^i(n)$ , the possibility operator shifts a world of evaluation so the modifying descriptive content would be taken from *a new world-parameter*. But in a narrow-scope reading,  $(\lambda x. \diamond Q(x))_n^i(n)$ , we take first a modifying descriptive content from a world of evaluation and then check if it is the case that the term's referent belongs to its extension. A term's descriptive content taken from different world-parameters could differ, so the predicates modified by it could be different and in effect it is possible for formulas with both a narrow and a wide scope to differ in truth-conditions. This is only a hypothetical possibility which cannot be proven yet because of a limitation of predicate abstracts which could be modified to predicates abstracted from atomic formulas and negations of atomic formulas. Nevertheless, it is possible to prove that formulas  $\sim (\lambda x. Q(x))_n^i(n)$  and  $(\lambda x. \sim Q(x))_n^i(n)$  with different scope of negation have no equal truth-conditions (see *Statement I* in Sect. 7). In the case of modified predicates,  $(\lambda x. \varphi)_{\lambda y. \psi}^i$ , we are talking about  $\varphi$  and a way of doing it  $\psi$ . But in case a predicate abstract is modified by a name-term  $n$ ,  $(\lambda x. \varphi)_n^i$ , we are not talking about *any particular way of doing  $\varphi$* . Consider:

<sup>17</sup> Compare Kripke's remark about rigidity and scopes of alethic modalities in (1980: 12 footnote 15). Fitting and Mendelsohn (1998: 217) characterize a term's rigidity as the equality of the broad scope to the narrow scope reading.

(5) Lukashenko is not blacklisted (as such)

We can write (5) either as (5') or as (5''):

$$(5') \quad (\lambda x. \sim V(x))_n(n)$$

$$(5'') \quad \sim (\lambda x. V(x))_n(n).$$

According to *Definition 7* (5') is true then there is a description from the set of descriptions associated with 'Lukashenko', for example 'the BOC chairman called [lukašenko]', such that it is true about Lukashenko that he is not blacklisted as the BOC chairman called [lukašenko]. As we know from the story in the beginning, (5') is true. In (5'') we deny that there is *any* description in the set of descriptions associated with 'Lukashenko', such that it is true about him that he is blacklisted *in any way*. But he is blacklisted as the President of Belarus called [lukašenko], so (5'') is false. It is important to notice that  $(\lambda x. \sim V(x))_n(n)$  and  $(\lambda x. V(x))_n(n)$  *could be true together*, that is why it is possible to simultaneously agree and disagree with (5) and remain consistent. In such cases such as this when we know that there is a description  $!y.[_j]\varphi$  associated with a proper name  $n$  such that  $d$ , the referent of  $n$ , is  $Q$  as  $\varphi$ ,  $d \in I(Q_{\lambda y.\varphi})$ , and we know that there is another description  $!y.[_j]\psi$  associated with the proper name such that  $d \notin I(Q_{\lambda y.\varphi})$ , we explicitly express this modifying content ('Paderewski is popular as a musician but he is not popular as a politician').

I am grateful to an anonymous referee for calling my attention to sentences with modified predicate of identity, 'to be identical as  $\varphi$ '. From a technical point of view, there is nothing in semantics presented here which restrains you from choosing any subset of extension of identity predicate as a representation of extension 'to be identical as  $\varphi$ ' predicate (e.g. you can choose a proper subset, an empty set or the whole set of pairs  $\langle d, d \rangle$  such that  $d$  is  $\varphi$ ). Could something, taken as  $\varphi$ , not be identical with itself (e.g. taken as  $\psi$ ) or not?—At this stage the notion needs further investigation and thus I leave it open. However, I think that an interpretation 'to be identical as being called  $\alpha$ ' is simply a set of pairs  $\langle d, d \rangle$  such that  $d$  is called  $\alpha$ , and that is why I share the referee's intuition that there is no true reading of sentences such as 'Muhammad Ali is not (identical with) Cassius Clay (as such)'. Let me show that there is no true reading of negated sentences with a predicate of identity modified by a descriptive content of a proper name (assuming that an interpretation of 'to be identical as being called  $\alpha$ ' is simply a set of pairs  $\langle d, d \rangle$  such that  $d$  is called  $\alpha$ ). As I mentioned earlier, sentences with predicates modified by a descriptive content of proper names and negation are ambiguous between two readings. One of the readings says that there is a description  $!x.[_j]\varphi$  in the set of descriptions associated with 'Muhammad Ali', such that it is true about Ali that he as  $\varphi$  is not identical with Cassius Clay. Formula  $\varphi$  is a conjunction called  $[muhammad\ ali](x) \wedge Q(x)$ , where  $Q$  is an atomic predicate true of Ali/Cassius. By definition modifiers are closed under conjunction so for 'Ali is not identical with Cassius Clay as being called  $[muhammad\ ali]$  and  $Q$ ,' to be true it should be so that Ali is not identical with Cassius Clay as being called  $[muhammad\ ali]$ . Yet this contradicts with our assumption—it is highly doubtful that an interpretation of predicates 'to be identical as being called  $\alpha$ ' differs from the set of pairs  $\langle d, d \rangle$  such that  $d$  is called  $\alpha$ . The other reading says that there is no description in the set of descriptions associated

with ‘Muhammad Ali’ such that it is true about Ali that he is identical with Cassius Clay *in any way*. This reading is clearly false for the same reason. Both readings emerge as being false which means that sentences of the form *NN is MM as such*, where NN and MM are coreferential names, are true according to this account (under the assumption that an interpretation of ‘to be identical as being called  $\alpha$ ’ is simply a set of pairs  $\langle d, d \rangle$  such that is called  $\alpha$ ).

## 6 Concluding remarks

I raise the hypothesis that sentences with proper names as  $[Name][Predicate]$  are ambiguous between two readings, (I) and (II),

- (I)  $[Name][Predicate]$ ,  
 (II)  $[Name] \underbrace{[Predicate]}_{\text{modified by } [Name]}$ .

From a pragmatic point of view readings (I) and (II) are non-equal. For example, using the criterion of relevance of an input to an individual from The Relevance Theory (Sperber and Wilson 1995: 265–266), we can predict that reading (I) should be a default reading. You need more effort to process (II)—you should additionally take into account circumstances of action expressed by modifying descriptive content—so (II) would be less relevant to you unless it has a greater cognitive effect overcoming the additional costs of processing. Consider for example:

(6) The papal nuncio supported an anarchist protest

What you understand as a default is that the papal nuncio supported an anarchist protest *as the papal nuncio*, because, read in this way, (6) has the effect of an information bomb (compared to the information that the papal nuncio supported the protest as a private person). I agree with Landman (1989: 741) that sentences have preferred readings with modified predicates when there is a (salient) semantic connection between meanings of a modifying predicate and a predicate being modified. Consider his example: ‘The chairman is well-paid’. Both predicates ‘being well-paid’ and ‘being a chairman’ are semantically related because both concern a job. The stronger the connection is, the easier it leads to substitution failure (comparative to sentences with less related predicates, e.g. ‘The chairman wants to eat meat’, 1989: 741).

Unlike definite descriptions, proper names are rigid designators and are not-sensitive to scope differences. When a proper name’s descriptive content is used only to pick up the reference and not to modify a predicate, there are no differences in operators’ scope. Consider:

- (7) Romain Gary won the Prix Goncourt in 1975  
 (8) Romain Gary was the only person who won the Prix Goncourt twice

Sentence (7) could be written as (7′) or as (7″) (I will use ‘w’ for ‘win’ and ‘n’ for ‘Romain Gary’):

- (7′)  $P(\lambda x.W(x))(n)$ —‘a certain person, Romain Gary, won the Prix Goncourt in 1975’;

(7'')  $\mathbf{P}(\lambda x.W(x))_n(n)$ —‘a certain person, Romain Gary, won the Prix Goncourt in 1975 (as Romain Gary)’

Formula (7') has exactly the same truth-conditions as formula  $(\lambda x.\mathbf{PW}(x))(n)$  because in both cases you check if the certain person, Romain Gary, won the Prix. He won, so (7') is true. But he won it not as Romain Gary, but as Émile Ajar. He is the only author to have won the Prix Goncourt twice. This prize for French language literature is awarded only once to an author and Gary, who had already received the prize in 1956, wrote a book as Emile Ajar to receive the prize again. So (7'') is false. In a similar way, if the sentence (8) is understood without modification, it is true (‘a certain person, Romain Gary, was the only person who won the Prix Goncourt twice’), but if it is understood as ‘a certain person, Romain Gary, won the Prix Goncourt twice *as Romain Gary*’) it is false.

The substitution of coreferential names in simple sentences could fail, because the different descriptive content of proper names modifies the main predicate differently, so in effect sentences could have different truth conditions. The double truth conditions of different readings (simple and modified) are responsible for the mixed intuitions which speakers feel about such examples. The raised hypothesis about the additional truth-conditional relevance of descriptive content associated with a proper name allows one to explain why speakers associate a descriptive content with a proper name—using the descriptive content as adjuncts, they could express propositions that could not be expressed in any other way (without *as*-phrases), for example ‘Romain Gary won the Prix Goncourt twice *but only once as Romain Gary*’.

In Appendix I will present a formal machinery [languages  $\mathcal{L}$  and  $\mathcal{L}^+$  (without and with name-terms)] and prove some useful statements.

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## Appendix: The formal representation of modified predicates

The languages  $\mathcal{L}$  and  $\mathcal{L}^+$  are based on first-order predicate logic with identity and descriptions (I followed Fitting and Mendelsohn 1998). I will skip all standard definitions and present the definitions that are specific for a formal representation of

modified predicates. Let me start from the language  $\mathcal{L}$  which contains only two sorts of terms: variables and iota-terms.

**Definition I** The alphabet of  $\mathcal{L}$ .

A first-order language  $\mathcal{L}$  contains the following symbols: sentential connectives  $\wedge, \vee, \rightarrow, \leftrightarrow, \sim$ ; quantifiers  $\exists, \forall$ ; an infinite set of individual variables  $x_1, x_2, x_3, \dots$ ; an infinite set of predicate constants  $P_1, P_2, P_3, \dots$ , with a positive integer (an arity) assigned to each of them; identity sign  $=$ ; the definite descriptions operator  $\iota$ ; the abstraction operator  $\lambda$ ; temporal operators of past **P** and future **F**; an infinite set of temporal operators  $[_i]$  ('true at  $t_i$ '), where  $i \in \mathbb{N}$ ; modal operators  $\Box, \Diamond$ ; an infinite set of distinguished predicate constants  $N_1, N_2, N_3, \dots$ ; a set of numerical symbols for natural numbers; the left parenthesis (, the right parenthesis ).

**Definition II** The syntax of  $\mathcal{L}$ .

Predicate constants and the predicate abstracts, modified atomic predicates and modified predicate abstracts defined below are predicates of  $\mathcal{L}$ . An *atomic predicate* of  $\mathcal{L}$  is any predicate constant. The notions of a formula, a term, a predicate and free variable occurrence are defined as follows:

the notions of a variable (R1), a predicate constant (R2), an atomic formula (R3),  $\sim \varphi$  (R4),  $(\varphi \wedge \psi)$ ,  $(\varphi \vee \psi)$ ,  $(\varphi \rightarrow \psi)$ ,  $(\varphi \leftrightarrow \psi)$  (R5), **P** $\varphi$ , **F** $\varphi$ ,  $[_i]\varphi$  (R6),  $\Box\varphi$ ,  $\Diamond\varphi$  (R7),  $\forall_x\varphi$ ,  $\exists_x\varphi$  (R8),  $\iota x.\varphi$  (R9),  $(\lambda x.\varphi)$  (R10) are defined in a standard way;

- R11. if  $Q$  is a 1-place predicate constant and  $x$  is a variable, then  $(\lambda x.Q(x))$  is a modifier. Modifiers contain no free variable occurrences;
- R12. if  $(\lambda x.\varphi)$ ,  $(\lambda x.\psi)$  are modifiers, then  $(\lambda x.(\varphi \wedge \psi))$  is a modifier;
- R13. if  $Q$  is a  $n$ -place predicate constant and  $(\lambda x.\varphi)$  is a modifier then  $Q_{\lambda x.\varphi}^i$  is  $n$ -place atomic predicate modified by  $(\lambda x.\varphi)$  on  $i$ th argument place of  $Q$  (where  $1 \leq i \leq n$ );
- R14. if  $(\lambda x.Q(z_1, \dots, z_n))$  is a predicate abstract and  $\lambda x.\psi$  is a modifier, then  $(\lambda x.Q(z_1, \dots, z_n))_{\lambda y.\psi}^i$  is a predicate abstract modified by  $(\lambda x.\psi)$  on  $i$ th argument place of  $Q$  (where  $1 \leq i \leq n$ ); the free variable occurrences in  $(\lambda x.Q(z_1, \dots, z_n))_{\lambda y.\psi}^i$  are those of  $(\lambda x.Q(z_1, \dots, z_n))$
- R15. if  $(\lambda x. \sim Q(z_1, \dots, z_n))$  is a predicate abstract and  $(\lambda x.\psi)$  is a modifier, then  $(\lambda x. \sim Q(z_1, \dots, z_n))_{\lambda y.\psi}^i$  is a predicate abstract modified by  $(\lambda x.\psi)$  on  $i$ th argument place of  $Q$  (where  $1 \leq i \leq n$ ); the free variable occurrences in  $(\lambda x. \sim Q(z_1, \dots, z_n))_{\lambda y.\psi}^i$  are those of  $(\lambda x.Q(z_1, \dots, z_n))$ ;
- R16. if  $Q$  is a  $n$ -place predicate constant,  $Q_{\lambda x.\varphi}^i$  is  $n$ -place modified predicate and  $z_1, \dots, z_n$  is an  $n$ -element sequence of variables, then  $Q_{\lambda x.\varphi}^i(z_1, \dots, z_n)$  is a formula in which all variable occurrences in the  $n$ -element sequence are free;
- R17. if  $(\lambda x.\varphi)$  is a predicate abstract and  $s$  is a term, then  $(\lambda x.\varphi)(s)$  is a formula; the free occurrences of variables in  $(\lambda x.\varphi)(s)$  are those of  $(\lambda x.\varphi)$  together with those of  $s$ ;



- R18. if  $(\lambda x.\varphi)_{\lambda y.\psi}^i$  is a modified predicate abstract and  $s$  is a term, then  $(\lambda x.\varphi)_{\lambda y.\psi}^i(s)$  is a formula; the free occurrences of variables in  $(\lambda x.\varphi)_{\lambda y.\psi}^i(s)$  are those of  $(\lambda x.\varphi)_{\lambda y.\psi}^i$  together with those of  $s$ ;
- R19. nothing else is a formula, a term, a predicate, a modifier and a free occurrence of a variable.

*Notational convention:*

- if  $Q$  is a 1-place predicate constant and  $\beta$  is a modifier, then instead of ' $Q_\beta^1$ ' we will write ' $Q_\beta$ ';
- if  $Q$  is a  $n$ -place predicate constant and  $(\lambda x.P(x))$  is a modifier, then instead of ' $Q_{\lambda x.P(x)}^i$ ' we will write ' $Q_P^i$ '.

### Definition III The semantics of $\mathcal{L}$ .

A varying domain first-order model  $\mathfrak{M}$  for  $\mathcal{L}$  is a structure  $\mathfrak{M} = \langle \mathcal{D}, T, <, W, I \rangle$ , such that:

- $\mathcal{D}$  is a domain function mapping pairs of possible world and time  $\langle w, t \rangle$  to non-empty sets. The domain of the model is the set  $\cup \{ \mathcal{D}_{\langle w, t \rangle} : w \in W, t \in T \}$ . We write  $\mathcal{D}_{\mathfrak{M}}$  for the domain of the model  $\mathfrak{M}$  and  $\mathcal{D}_{\langle w, t \rangle}$  for a value of the function  $\mathcal{D}$  for an argument  $\langle w, t \rangle$ ;
  - $T$  is a set of natural numbers and  $<$  ('earlier then') is a linear order defined on elements of  $T$  (a set  $(T, <)$  is thought as a flow of time);
  - $W$  is a non-empty set of possible worlds;
  - $I$  is a function which assigns an extension to each pair of an atomic predicate or modified atomic predicate of  $\mathcal{L}$  and a pair  $\langle w, t \rangle$ , where  $w \in W, t \in T$ , in the following way:
    - if  $Q$  is a  $n$ -place predicate constant, then  $I_{\langle w, t \rangle}(Q) \subseteq \mathcal{D}_{\mathfrak{M}}^n$ ;
    - $I_{\langle w, t \rangle}(=) = \{ \langle d, d \rangle \in \mathcal{D}_{\mathfrak{M}} \}$ ;
- let  $g$  be a variable assignment (a mapping that assigns to each free variable  $x$  some member  $g(x)$  of the model domain  $\mathcal{D}_{\mathfrak{M}}$ ) and let  $I_{\langle w, t \rangle}^g$  be a function which assigns an extension to each pair of an atomic predicate, a modified predicate or a term of  $\mathcal{L}$  and a pair  $\langle w, t \rangle$ , where  $w \in W, t \in T$ , in the following way:
- if  $x$  a variable, then  $I_{\langle w, t \rangle}^g(x) = g(x)$  for any  $\langle w, t \rangle$ ;
  - $I \subseteq I^g$  for any  $g$ ;

the notion of interpretation of terms other than variables and interpretation of modified predicates and satisfaction of formulas in  $\mathfrak{M}$  are defined as follows:

- S1. if  $Q$  is a  $n$ -place predicate constant and  $y_1, \dots, y_n$  are variables, then  $\mathfrak{M}^{gwt} \models Q(y_1, \dots, y_n)$  iff  $\langle g(y_1), \dots, g(y_n) \rangle \in I_{\langle w, t \rangle}(Q)$ ;

the notions of satisfaction of  $\sim \varphi$  (S2),  $(\varphi \wedge \psi)$  (S3),  $(\varphi \vee \psi)$  (S4),  $(\varphi \rightarrow \psi)$  (S5),  $(\varphi \leftrightarrow \psi)$  (S6) are defined in a standard way;

- S7. if  $Q$  is a  $n$ -place predicate constant,  $P$  is a 1-place predicate constant and  $x$  is a variable, then  $I_{\langle w, t \rangle} \left( Q_{\lambda x. P(x)}^i \right) \in \mathcal{P}(\{ \langle d_1, \dots, d_i, \dots, d_n \rangle \in I_{\langle w, t \rangle}(Q) : d_i \in I_{\langle w, t \rangle}(P) \})$ ;
- S8. if  $Q_{\lambda x. P(x)}^i, Q_{\lambda y. P(y)}^i$  are  $n$ -place atomic predicates modified by  $\lambda x. P(x), \lambda y. P(y)$  on  $i$ th argument place and  $x, y$  are variables, then  $I_{\langle w, t \rangle} \left( Q_{\lambda x. P(x)}^i \right) = I_{\langle w, t \rangle} \left( Q_{\lambda y. P(y)}^i \right)$ ;
- S9. if  $Q$  is a  $n$ -place predicate constant,  $x$  is a variable, and  $(\lambda x. \varphi), (\lambda x. \psi)$  are modifiers, then  $I_{\langle w, t \rangle} \left( Q_{\lambda x. (\varphi \wedge \psi)}^i \right) = I_{\langle w, t \rangle} \left( Q_{\lambda x. \varphi}^i \right) \cap I_{\langle w, t \rangle} \left( Q_{\lambda x. \psi}^i \right)$ ;
- S10. if  $Q(z_1, \dots, z_n)$  is an atomic formula and  $(\lambda x. Q(z_1, \dots, z_n))_{\lambda y. \psi}^i$  is a modified predicate abstract, then  $I_{\langle w, t \rangle}^g \left( (\lambda x. Q(z_1, \dots, z_n))_{\lambda y. \psi}^i \right) = \left\{ d \in \mathcal{D}_{\mathfrak{M}} : \mathfrak{M}^g \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)^{wt} \models Q_{\lambda y. \psi}^i(z_1, \dots, z_n) \right\}$ ;
- S11. if  $\sim Q(z_1, \dots, z_n)$  is a negation of an atomic formula and  $(\lambda x. \sim Q(z_1, \dots, z_n))_{\lambda y. \psi}^i$  is a modified predicate abstract, then  $I_{\langle w, t \rangle}^g \left( (\lambda x. \sim Q(z_1, \dots, z_n))_{\lambda y. \psi}^i \right) = \left\{ d \in \mathcal{D}_{\mathfrak{M}} : \mathfrak{M}^g \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)^{wt} \not\models Q_{\lambda y. \psi}^i(z_1, \dots, z_n) \right\}$ ;
- S12. if  $Q$  is a  $n$ -place predicate constant,  $(\lambda x. \varphi)$  is a modifier and  $Q_{\lambda x. \varphi}^i$  is a  $n$ -place modified predicate, then  $\mathfrak{M}^{gwt} \models Q_{\lambda x. \varphi}^i(z_1, \dots, z_n)$  iff  $\langle g(z_1), \dots, g(z_n) \rangle \in I_{\langle w, t \rangle} \left( Q_{\lambda x. \varphi}^i \right)$ ;

the notions of satisfaction  $\mathbf{P}\varphi$  (S13),  $\mathbf{F}\varphi$  (S14) are defined in a standard way;

- S15. if  $\varphi$  is a formula, then  $\mathfrak{M}^{gwt_i} \models_{[i]} \varphi$  iff  $\mathfrak{M}^{gwt_i} \models \varphi$ ;

the notions of satisfaction  $\Box\varphi$  (S16),  $\Diamond\varphi$  (S17),  $\forall_x\varphi$  (S18),  $\exists_x\varphi$  (S19) are defined in a standard way;

- S20. if  $\mathfrak{M}^g \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)^{wt} \models \varphi$  for exactly one  $d \in \mathcal{D}_{\mathfrak{M}}$ , then  $I_{\langle w, t \rangle}^g(\iota x. \varphi) = d$ ; if it is not the case that  $\mathfrak{M}^g \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)^{wt} \models \varphi$  for exactly one  $d \in \mathcal{D}_{\mathfrak{M}}$ , then  $\iota x. \varphi$  fails to designate at  $\langle w, t \rangle$  in  $\mathfrak{M}$  with respect to  $g$ ;

the notion of satisfaction of  $(\lambda x. \varphi)(s)$  (S21) is defined in a standard way;

- S22. if a term  $s$  designates at  $\langle w, t \rangle$  in  $\mathfrak{M}$  with respect to  $g$  and  $(\lambda x. \varphi)_{\lambda y. \psi}^i$  is a modified predicate abstract, then  $\mathfrak{M}^{gwt} \models (\lambda x. \varphi)_{\lambda y. \psi}^i(s)$  iff  $I_{\langle w, t \rangle}^g(s) \in$

$I_{\langle w, t \rangle}^g((\lambda x. \varphi)_{\lambda y. \psi}^i)$ ; if a term  $s$  fails to designate at  $\langle w, t \rangle$  in  $\mathfrak{M}$  with respect to  $g$ , then  $\mathfrak{M}^{gwt} \not\models (\lambda x. \varphi)_{\lambda y. \psi}^i(s)$ .

I will use symbol ' $\lambda x. \varphi$ ' for a special case of  $\lambda x. \varphi$  terms with only one variable  $x$  which occurs free in  $\varphi$ . There are no free variable occurrences in  $\lambda x. \varphi$  and due to this if  $I_{\langle w, t \rangle}^g(\lambda x. \varphi)$  is defined then  $I_{\langle w, t \rangle}^g(\lambda x. \varphi) = I_{\langle w, t \rangle}^{g'}(\lambda x. \varphi)$  for any assignments  $g$  and  $g'$ . That is why instead of ' $I_{\langle w, t \rangle}^g(\lambda x. \varphi)$ ' we will write ' $I_{\langle w, t \rangle}(\lambda x. \varphi)$ ' which should be understood as ' $I_{\langle w, t \rangle}^g(\lambda x. \varphi)$ ' where  $g$  is any assignment.

Now I will expand language  $\mathcal{L}$  to  $\mathcal{L}^+$  by adding name-terms. I will skip all syntactical and semantic definitions of  $\mathcal{L}^+$  duplicating the definitions of  $\mathcal{L}$  and will write below only new ones.

**Definition IV** The alphabet of  $\mathcal{L}^+$ .

A first-order language  $\mathcal{L}^+$  contains all symbols of  $\mathcal{L}$  with the addition of an infinite set of name-terms  $\mathcal{N} = \{n_1, n_2, n_3, \dots\}$ .

**Definition V** The syntax of  $\mathcal{L}^+$ .

- R1. the same as R1. of  $\mathcal{L}$ ;
- R2. a name-term  $n_i$  is a term with no free variable occurrences;
- R3.–R12. are the same as R2.–R11. of  $\mathcal{L}$ ;
- R13.  $n_i$  is a modifier, where  $n_i$  is a name-term;
- R14.–R16. are the same as R12.–R14. of  $\mathcal{L}$ ;
- R17. if  $((\lambda x. Q(z_1, \dots, z_n))$  is a predicate abstract and  $n_j$  is a name-term, then  $(\lambda x. Q(z_1, \dots, z_n))_{n_j}^i$  is a predicate abstract modified by  $n_j$  on  $i$ th argument place of  $Q$  (where  $1 \leq i \leq n$ ); the free variable occurrences in  $(\lambda x. Q(z_1, \dots, z_n))_{n_j}^i$  are those of  $((\lambda x. Q(z_1, \dots, z_n))$ ;
- R18. if  $(\lambda x. \sim Q(z_1, \dots, z_n))$  is a predicate abstract and  $n_j$  is a name-term, then  $(\lambda x. \sim Q(z_1, \dots, z_n))_{n_j}^i$  is a predicate abstract modified by  $n_j$  on  $i$ th argument place of  $Q$  (where  $1 \leq i \leq n$ ); the free variable occurrences in  $(\lambda x. \sim Q(z_1, \dots, z_n))_{n_j}^i$  are those of  $(\lambda x. \sim Q(z_1, \dots, z_n))$ ;
- R19.–R21. are the same as R16.–R18. of  $\mathcal{L}$ ;
- R22. if  $(\lambda x. \varphi)_{n_j}^i$  is a modified predicate abstract and  $n_k$  is a name-term, then  $(\lambda x. \varphi)_{n_j}^i(n_k)$  is a formula iff  $k = j$ ; the free variable occurrences in  $(\lambda x. \varphi)_{n_j}^i(n_k)$  are those of  $(\lambda x. \varphi)$ ;
- R23. the same as R19. of  $\mathcal{L}$

**Definition VI** The semantics of  $\mathcal{L}^+$

Let  $\mathfrak{M} = \langle \mathcal{D}, T, <, W, I \rangle$  be a model of  $\mathcal{L}$ . A varying domain first-order model  $\mathfrak{M}^{\leq}$  for  $\mathcal{L}^+$  is a structure  $\mathfrak{M}^{\leq} = \langle \mathcal{D}, T, <, W, I^{\leq} \rangle$ , where  $I^{\leq} \upharpoonright \mathcal{L} = I$ .

Using already defined properties of  $\mathfrak{M}$  (*Definition III*) we define the following sets, relations and functions.

*S(a): set*  $\Gamma_{\mathcal{L}}$

Set  $\Gamma_{\mathcal{L}}$  is a set of iota-terms  $!x.[_i]\varphi$  of  $\mathcal{L}$ .  $!x.[_i]\varphi \in \Gamma_{\mathcal{L}}$  iff 1) there is a world  $w \in W$  such that for every time  $t \in T$   $!x.[_i]\varphi$  designates at  $\langle w, t \rangle$  in  $\mathfrak{M}$ ; 2)  $\varphi = (N_i(x) \wedge Q(x))$  where  $N_i$  is a distinguished predicate and  $Q$  is a 1-place undistinguished predicate. (I will use symbols ' $\gamma_i$ ', ' $\gamma_j$ ', for members of  $\Gamma_{\mathcal{L}}$ ).

*S(b): set*  $\Delta$

$\Delta \subseteq \Gamma_{\mathcal{L}} \times W$ .  $\langle \gamma_i, w \rangle \in \Delta$  iff for any time  $t \in T$   $I_{\langle w, t \rangle}(\gamma_i)$  is defined.

*S(c): relation*  $\mathbb{R}$

$\mathbb{R} \subseteq \Delta^2$ .  $\langle \gamma_i, w \rangle \mathbb{R} \langle \gamma_j, w' \rangle$  iff for any time  $t \in T$   $I_{\langle w, t \rangle}(\gamma_i) = I_{\langle w', t \rangle}(\gamma_j)$  and there is the same predicate  $N_k$  in  $\gamma_i, \gamma_j$ .

Let  $\Delta/\mathbb{R}$  be a partition of set  $\Delta$  by equivalence relation  $\mathbb{R}$  and  $[\langle \gamma_i, w \rangle]_{\mathbb{R}}$  be an equivalence class from  $\Delta/\mathbb{R}$ .

*S(d): function*  $\mathbb{F}$

$\mathbb{F} : \Delta/\mathbb{R} \rightarrow \mathcal{D}_{\mathfrak{M}}$ . For any  $[\langle \gamma_i, w \rangle]_{\mathbb{R}} \in \Delta/\mathbb{R}$ ,  $\mathbb{F}([\langle \gamma_i, w \rangle]_{\mathbb{R}}) = d$ , where for any time  $t \in T$   $d = I_{\langle w, t \rangle}(\gamma_j)$  for any  $\langle \gamma_i, w \rangle \in [\langle \gamma_j, w \rangle]_{\mathbb{R}}$ .

Let  $\leq$  be any well-order relation on a set  $\Delta/\mathbb{R}$  and let  $\langle \Delta/\mathbb{R}, \leq \rangle$  be well-ordered set.

*S(e): function*  $\mathbb{Q}^{\leq}$

$\mathbb{Q}^{\leq} : \mathcal{N} \rightarrow \Delta/\mathbb{R}$ . Function  $\mathbb{Q}^{\leq}$  for an argument  $n_i$  gives an equivalence class  $[\langle \gamma_i, w \rangle]_{\mathbb{R}}$  in the following way:

- for  $n_1$   $\mathbb{Q}^{\leq}$  gives the least element of  $\langle \Delta/\mathbb{R}, \leq \rangle$ ;
- for every next element of  $\mathcal{N}$  (with respect to an index)  $\mathbb{Q}^{\leq}$  gives next element of  $\langle \Delta/\mathbb{R}, \leq \rangle$ ;
- in case there are no next element in  $\langle \Delta/\mathbb{R}, \leq \rangle$ , then for a next element of  $\mathcal{N}$   $\mathbb{Q}^{\leq}$  gives the least element of  $\langle \Delta/\mathbb{R}, \leq \rangle$ .

*S(f): relation*  $\mathbb{S}$

$\mathbb{S} \subseteq \Delta^2$ .  $\langle \gamma_i, w \rangle \mathbb{S} \langle \gamma_j, w' \rangle$  iff  $\langle \gamma_i, w \rangle, \langle \gamma_j, w' \rangle$  belong to the same equivalence class  $[\langle \gamma_i, w \rangle]_{\mathbb{R}}$  and  $w = w'$ .

*S(g): function*  $h^{\leq}$

$h^{\leq} : \mathcal{N} \times W \rightarrow \Delta/\mathbb{S}$ . For any  $n_i \in \mathcal{N}, w \in W$   $h^{\leq}(n_i, w) = [\langle \gamma_j, w \rangle]_{\mathbb{S}} \subseteq \mathbb{Q}^{\leq}(n_i)$  if there is such equivalence class, otherwise  $h^{\leq}(n_i, w)$  is undefined.

Semantic rules *S1.*–*S20.* of language  $\mathcal{L}^+$  are the same as rules *S1.*–*S20.* of language  $\mathcal{L}$  (except of talking about  $I^{\leq}$  instead of  $I$ );

*S21.* if  $n_i$  is a name-term and  $\Gamma_{\mathcal{L}} \neq \emptyset$ , then  $I_{\langle w, t \rangle}^{\leq}(n_i) = \mathbb{F}(\mathbb{Q}^{\leq}(n_i))$ ; if  $\Gamma_{\mathcal{L}} = \emptyset$ , then  $n_i$  fails to designate in  $\mathfrak{M}^{\leq}$  (at any  $\langle w', t' \rangle$ );

- S22. if a term  $s$  designates at  $\langle w, t \rangle$  in  $\mathfrak{M}^{\leq}$  with respect to  $g$ , then  $\mathfrak{M}^{\leq gwt} \models (\lambda x. \varphi)(s)$  iff  $\mathfrak{M}^{\leq g} \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)_{wt} \models \varphi$ , where  $d = I_{\langle w, t \rangle}^{\leq g}(s)$ ; if a term  $s$  fails to designate at  $\langle w, t \rangle$  in  $\mathfrak{M}^{\leq}$  with respect to  $g$ , then  $\mathfrak{M}^{\leq gwt} \not\models (\lambda x. \varphi)(s)$ ;
- S23. if a term  $s$  designates at  $\langle w, t \rangle$  in  $\mathfrak{M}^{\leq}$  with respect to  $g$  and  $((\lambda x. \varphi)_{\lambda y. \psi}^i)$  is a modified predicate abstract, then  $\mathfrak{M}^{\leq gwt} \models (\lambda x. \varphi)_{\lambda y. \psi}^i(s)$  iff  $I_{\langle w, t \rangle}^{\leq g}(s) \in I_{\langle w, t \rangle}^{\leq g}((\lambda x. \varphi)_{\lambda y. \psi}^i)$ ; if a term  $s$  fails to designate at  $\langle w, t \rangle$  in  $\mathfrak{M}^{\leq}$  with respect to  $g$ , then  $\mathfrak{M}^{\leq gwt} \not\models (\lambda x. \varphi)_{\lambda y. \psi}^i(s)$ ;
- S24. if  $n_k$  is a name-term and  $(\lambda x. \varphi)_{n_k}^i$  is a predicate abstract modified by  $n_k$ , then  $\mathfrak{M}^{\leq gwt} \models (\lambda x. \varphi)_{n_k}^i(n_k)$  iff there is a description  $!y. [j] \psi \in \pi_1(h^{\leq}(n_k, w))$ , such that  $\mathfrak{M}^{\leq gwt} \models (\lambda x. \varphi)_{\lambda y. \psi}^i(n_k)$ .

**Theorem I** (taking a modifier ‘in and out’ of a predicate abstract) *Let  $s$  be any term,  $Q(z_1, \dots, z_n)$  any atomic formula,  $\lambda y. \psi$  any modifier,  $\langle w, t \rangle$  any evaluation point,  $g$  any variable assignment and  $\mathfrak{M}^{\leq}$  any model. Then:*

$$\mathfrak{M}^{\leq gwt} \models (\lambda x. Q_{\lambda y. \psi}^i(z_1, \dots, z_n))(s) \text{ iff } \mathfrak{M}^{\leq gwt} \models (\lambda x. Q(z_1, \dots, z_n))_{\lambda y. \psi}^i(s).$$

*Proof* If  $s$  fails to designate in  $\mathfrak{M}^{\leq}$  at  $\langle w, t \rangle$  with respect to  $g$ , then (Def. VI.S22, S23) both formulas are not satisfied in  $\mathfrak{M}^{\leq}$  at  $\langle w, t \rangle$  with respect to  $g$ . Let us assume that  $s$  designates in  $\mathfrak{M}^{\leq}$  at  $\langle w, t \rangle$  with respect to  $g$ .

$$\mathfrak{M}^{\leq gwt} \models (\lambda x. Q_{\lambda y. \psi}^i(z_1, \dots, z_n))(s) \text{ iff (Def. VI.S22) } \mathfrak{M}^{\leq g} \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)_{wt} \models Q_{\lambda y. \psi}^i(z_1, \dots, z_n), \text{ where } d = I_{\langle w, t \rangle}^{\leq g}(s) \text{ iff (Def. VI.S10) } d \in I_{\langle w, t \rangle}^{\leq g}((\lambda x. Q(z_1, \dots, z_n))_{\lambda y. \psi}^i). \text{ Because } d = I_{\langle w, t \rangle}^{\leq g}(s), \text{ this is so iff (Def. VI.S23) } \mathfrak{M}^{\leq gwt} \models (\lambda x. Q(z_1, \dots, z_n))_{\lambda y. \psi}^i(s). \quad \square$$

**Theorem II** (taking a modifier ‘in and out’ of a predicate abstracted from negated atomic formula) *Let  $s$  be any term,  $\sim Q(z_1, \dots, z_n)$  any negated atomic formula,  $\lambda y. \psi$  any modifier,  $\langle w, t \rangle$  any evaluation point,  $g$  any variable assignment and  $\mathfrak{M}^{\leq}$  any model. Then:*

$$\mathfrak{M}^{\leq gwt} \models (\lambda x. \sim Q_{\lambda y. \psi}^i(z_1, \dots, z_n))(s) \text{ iff } \mathfrak{M}^{\leq gwt} \models (\lambda x. \sim Q(z_1, \dots, z_n))_{\lambda y. \psi}^i(s).$$

*Proof* Analogously as in Theorem I.  $\square$

### Statement I (difference in negation scopes)

There is a model  $\mathfrak{M}^{\leq}$ , a point of evaluation  $\langle w, t \rangle$ , a variable assignment  $g$ , a name-term  $n$  and a predicate constant  $Q$  such that

$$\mathfrak{M}^{\leq gwt} \models (\lambda x. \sim Q(z_1, \dots, z_m))_n^i(n) \text{ and } \mathfrak{M}^{\leq gwt} \not\models \sim (\lambda x. Q(z_1, \dots, z_m))_n^i(n).$$

*Proof* Let  $\mathfrak{M}^{\leq}$  be a model of  $\mathcal{L}^+$ ,  $W = \{w\}$ ,  $\mathcal{D}_{\langle w, t_1 \rangle} = \{\star, \odot\}$ ,  $\mathcal{D}_{\langle w, t_i \rangle} = \emptyset$  for  $i \neq 1$ . Let us use symbols ‘ $P$ ’, ‘ $S$ ’, ‘ $R$ ’ instead ‘ $P_1$ ’, ‘ $P_2$ ’, ‘ $P_3$ ’ of  $\mathcal{L}^+$ . Let  $I^{\leq}$  be defined in the following way:

	$P$	$S$	$R$	$P_i$ $i \neq 1, 2, 3$	$N_1$	$N_i$ $i \neq 1$	$P_S$	$P_R$	$P_{N_1}$
$I_{\langle w, t_1 \rangle}^{\leq}$	$\star$	$\star$	$\star$	$\emptyset$	$\star$	$\emptyset$	$\star$	$\emptyset$	$\star$
$I_{\langle w, t_i \rangle}^{\leq}$ $i \neq 1$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Let  $I_{\langle w, t \rangle}^{\leq}(Q_Q) = I_{\langle w, t \rangle}^{\leq}(Q)$  for any predicate constant  $Q$  and any  $\langle w, t \rangle$ . For every predicate not mentioned above and any  $\langle w, t \rangle$  function  $I^{\leq}$  gives  $\emptyset$ . According to definitions (Def. VI.S(a)–S(c), S(f)),  $\Gamma_{\mathcal{L}} = \{\gamma_1, \gamma_2, \gamma_3\}$ ,  $\Delta = \Delta/\mathbb{R} = \Delta/\mathbb{S}$ :

$\gamma_1 \mid !x.[_1](P(x) \wedge N_1(x))$	$I_{\langle w, t \rangle}^{\leq}$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$\Delta = \Delta/\mathbb{R} = \Delta/\mathbb{S}$ $\langle \gamma_1, w \rangle$ $\langle \gamma_2, w \rangle$ $\langle \gamma_3, w \rangle$
$\gamma_2 \mid !x.[_1](S(x) \wedge N_1(x))$		$\star$	$\star$	$\star$	
$\gamma_3 \mid !x.[_1](R(x) \wedge N_1(x))$					

Let  $\mathcal{L}^+$  contain a name-term  $n$ ;  $I_{\langle w, t \rangle}^{\leq}(n) = \star$  for any  $\langle w, t \rangle$ . Let us prove that

(1)  $\mathfrak{M}^{\leq gwt_1} \models (\lambda x. \sim P(x))_n(n)$  and (2)  $\mathfrak{M}^{\leq gwt_1} \not\models \sim (\lambda x. P(x))_n(n)$ .

(1)  $\mathfrak{M}^{\leq gwt_1} \models (\lambda x. \sim P(x))_n(n)$  iff (Def.VI.S24) there is a description

$!y.[_1]\psi \in \pi_1(h^{\leq}(n, w))$ , such that  $\mathfrak{M}^{\leq gwt_1} \models (\lambda x. \sim P(x))_{\lambda y. \psi}(n)$  iff (Theorem II)

there is a description  $!y.[_1]\psi \in \pi_1(h^{\leq}(n, w))$ , such that  $\mathfrak{M}^{\leq gwt_1} \models (\lambda x. \sim P_{\lambda y. \psi}(x))$

(n) iff (Def.VI.S22) there is a description  $!y.[_1]\psi \in \pi_1(h^{\leq}(n, w))$ , such that

$\mathfrak{M}^{\leq g} \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)^{wt_1} \models \sim P_{\lambda y. \psi}(x)$ , where  $d = I_{\langle w, t_1 \rangle}^{\leq}(n)$  iff (Def. VI.S2) there is a

description  $!y.[_1]\psi \in \pi_1(h^{\leq}(n, w))$ , such that  $\mathfrak{M}^{\leq g} \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)^{wt_1} \not\models P_{\lambda y. \psi}(x)$ , where  $d =$

$I_{\langle w, t_1 \rangle}^{\leq}(n)$  iff (Def.VI.S12) there is a description  $!y.[_1]\psi \in \pi_1(h^{\leq}(n, w))$ , such that

$d \notin I_{\langle w, t_1 \rangle}^{\leq}(P_{\lambda y. \psi})$ , where  $d = I_{\langle w, t_1 \rangle}^{\leq}(n)$ .

$d = I_{\langle w, t_1 \rangle}^{\leq}(n) = \star$ . Let a description  $!y.[_1]\psi$  be description  $\gamma_3, !x.[_1](R(x) \wedge N_1(x))$ .  $\star \notin I_{\langle w, t_1 \rangle}^{\leq}(P_R)$  so (Def.VI.S9)  $\star \notin I_{\langle w, t_1 \rangle}^{\leq}(P_{\lambda y. (R(y) \wedge N_1(y))})$ . This means that (1) is true.

(2)  $\mathfrak{M}^{\leq gwt_1} \models \sim (\lambda x. P(x))_n(n)$  iff (Def.VI.S2)

$\mathfrak{M}^{\leq gwt_1} \not\models (\lambda x. P(x))_n(n)$  iff there is no a description  $!y.[_1]\psi \in \pi_1(h^{\leq}(n, w))$ , such

that  $\mathfrak{M}^{\leq gwt_1} \models (\lambda x. P(x))_{\lambda y. \psi}(n)$  iff (Theorem II) there is no description

$!y.[_1]\psi \in \pi_1(h^{\leq}(n, w))$ , such that  $\mathfrak{M}^{\leq gwt_1} \models (\lambda x. P_{\lambda y. \psi}(x))(n)$  iff (Def.VI.S22) there

is no description  $!y.[_1]\psi \in \pi_1(h^{\leq}(n, w))$ , such that  $\mathfrak{M}^{\leq g} \left( \begin{smallmatrix} d \\ x \end{smallmatrix} \right)^{wt_1} \models P_{\lambda y. \psi}(x)$ , where

$d = I_{\langle w, t_1 \rangle}^{\leq}(n)$  iff (Def.VI.S12) there is no description  $!y.[\_]\psi \in \pi_1(h^{\leq}(n, w))$ , such that  $d \in I_{\langle w, t_1 \rangle}^{\leq}(P_{!y.\psi})$ , where  $d = I_{\langle w, t_1 \rangle}^{\leq}(n)$ .

$d = I_{\langle w, t_1 \rangle}^{\leq}(n) = *$ .  $*$   $\in I_{\langle w, t_1 \rangle}^{\leq}(P_S)$  and  $*$   $\in I_{\langle w, t_1 \rangle}^{\leq}(P_{N_1})$ , so (Def.VI.S9)  $*$   $\in I_{\langle w, t_1 \rangle}^{\leq}(P_{\lambda y.(S(y) \wedge N_1(y))})$ . This means that (Def.VI.S8)  $*$   $\in I_{\langle w, t_1 \rangle}^{\leq}(P_{\lambda x.(S(x) \wedge N_1(x))})$ . Description  $!x.[\_](S(x) \wedge N_1(x)) \in \pi_1(h^{\leq}(n, w))$ , so (2) is false.

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